National Certificate of Educational Achievement

2012 Assessment Report

Mathematics with Calculus Level 3
Statistics and Modelling Level 3

90635 Differentiate functions and use derivatives to solve problems

90636 Integrate functions and use integrals to solve problems

90638 Manipulate real and complex numbers, and solve problems

90639 Sketch graphs of conic sections and write equations related to conic sections

90642 Calculate confidence intervals for population parameters

90643 Solve straightforward problems involving probability

90644 Solve equations

90646 Use probability distribution models to solve straightforward problems

COMMENTARY

This was the final year for examinations to assess these achievement standards.

The 2012 papers followed the format of the three previous years. Each paper consisted of two multiple part questions with each question providing the opportunity for all grades of performance.

Above all else the defining characteristics of successful candidates were good algebraic skills. There was also a noticeable correlation between the ability of students to present their thoughts in a clear, logical manner and the attainment of grades above Achieved.

As usual the ability to differentiate (or integrate) alone was not sufficient to pass the differentiation (or integration) standards. Algebraic manipulation of expressions into forms appropriate for differentiating or integrating is vital. It is also essential in solving a problem once the differentiation or integration step has been carried out.

As in 2012 there were fewer students who attempted to answer questions using graphics calculators without first doing the required analytical steps. This is encouraging and demonstrates improved understanding of what is required.

STANDARD REPORTS

90635 Differentiate functions and use derivatives to solve problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- differentiated given functions using the chain rule, product rule or quotient rule as appropriate
- solved a problem in kinematics requiring differentiation
- found the equation of the normal to the graph of a given function for a specified value of x.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- did not use brackets when required in mathematical expressions
- differentiated given functions but did not produce sufficient evidence of being able to apply differentiation to solve problems (the standard requires both), or, less frequently, applied differentiation to solve problems but made errors in differentiating given functions
- found the equation of the tangent to the graph of a function rather than the equation of the normal as required.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- differentiated a function given parametrically
- identified points on the graph of a piecewise function at which the function had specified features related to calculus, and associated the derivative or second derivative with features of the graph (e.g. concavity)
- found a rate of change in a situation in which they first had to find a parameter for the problem (although the nature of the context should have been familiar to candidates, the nature of the problem would probably have been unfamiliar)
- differentiated a relation implicitly to find the gradient of the graph of the relation at a specified point on the graph.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- solved complex problems involving derivatives
- worked systematically and logically
- showed their reasoning clearly
- used mathematical symbolism and terminology correctly.

OTHER COMMENTS

Candidates displayed reasonable differentiation skills, but often came up short in the solving problems criteria. There are still a number of candidates who struggle with questions of the 1 (e) type - limits, continuity and differentiability. This type of problem is conceptual rather than algorithmic and is a good test of the understanding of some of the underlying ideas of calculus.

90636 Integrate functions and use integrals to solve problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- applied Simpson’s rule accurately
- correctly integrated square root functions and 1/x
- integrated trig functions such as sin 2x
- solved basic second order differential equations
- rearranged basic quotients into two parts and integrated each part
- solved simple differential equations using given information to find particular solutions.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- had difficulty with algebraic manipulation of rational expressions or 1/x
- were inaccurate with negative signs
- were not accurate when using a calculator
- incorrectly calculated the value of h for the Simpson's rule
- did not find the value of constants when solving simple differential equations
- used the Trapezium rule instead of Simpson's rule.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- understood how to find an area between two curves
- integrated complex differential equations by separating the variables correctly
- accurately integrated rational expressions
- integrated by substitution correctly, in particular changing from dx to du.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- were able to form and solve a complex differential equation in context where they are required to separate variables to integrate
- recognised that $(e^y)^2 = e^{2y}$ and integrated it correctly
- displayed the ability to accurately follow through a long sequence of algebraic steps
- rearranged the equation of a curve to have x as the subject instead of y
- were able to find the volume of revolution about a line parallel to the y-axis.

OTHER COMMENTS

While a number of minor arithmetic errors were ignored, candidates must remember that this is a mathematics standard and accurate calculations are expected.

90638 Manipulate real and complex numbers, and solve problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- solved quadratic equations by completing the square or using the quadratic formula and also simplified their answer leaving it in its simplest exact form
- solved an index equation by converting to the same base and equating indices or by taking the log of each side and rearranging to make *x* the subject
- solved an equation with logarithms by applying one of the log rules, converting to index form and rearranging to make *x* the subject

- simplified a power of a complex number either with a graphic calculator or by converting to polar form, applying de Moivre's theorem and writing the answer in its exact form
- solved a problem involving equal roots knowing that the discriminant had to equal zero and therefore solving the resulting equation with both of its solutions
- demonstrated the ability to use their graphic calculator well.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- could not give their answers as exact solutions
- could not use the quadratic formula and discriminant correctly
- could not write surds in their simplest form
- made careless errors when rearranging equations
- did not make *x* the subject of the equation as instructed
- could not accurately apply the rules of logarithms
- could not find the argument of a fourth quadrant complex number
- could not find an exact solution for a power of a complex number
- did not recognise that the equal roots question was asking them to recall their knowledge about quadratic theory and the discriminant.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- converted a complex number involving a parameter to polar form accurately. It was evident that many candidates knew what to do when they had converted but they routinely, incorrectly used –p and an angle of 0
- applied their knowledge of quadratic equations and their roots to solve a problem, using a variety of methods – quadratic formula, sum and product of roots as well as letting the roots be a and 2a, forming an equation and equating coefficients
- understood the meaning of the modulus signs
- simplified the quotient to 1+i and then demonstrated that the modulus was $\sqrt{2}$
- used $|z| = \sqrt{z.\overline{z}}$ to calculate the modulus
- used $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ to show that $\left|\frac{3+i}{2-1}\right| = \sqrt{2}$
- converted each of the numerator and denominator into polar form, divided the two complex numbers in polar form by dividing the moduli and subtracting the arguments to obtain $\sqrt{2}cis\frac{\pi}{4}$ and then using r equal to the modulus
- solved a more complicated index equation by rearranging, correctly taking the natural logarithm of each side and then rearranging again to make *x* the subject. Or alternatively other students started by factorising, noted that $e^x \neq 0$ and solved the remaining equation by taking the natural logarithm of each side and then rearranging again to make *x* the subject.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- showed clear, logical and valid mathematical steps in either of the proofs.
- recognised that $|z| = 1$ meant that r = 1 and used this information to re write the general form of $z = rcis\theta$ in the form: $z = cis\theta$
- used the even and odd trigonometric function properties: $\cos(-n\theta) = \cos n\theta$ and $\sin(-n\theta) = -\sin(n\theta)$ to write a simplified form of $z^n + z^{-n}$
- recognised that $cisn\theta$ and $cis(-n\theta)$ were conjugates and used their geometry to prove the required result.
- found the modulus $|z-a|^2 = (x-a)^2 + y^2$ as well as correctly factorising $b^2 - a^2$.

OTHER COMMENTS

A significant number of students did not understand what was meant by an exact value and gave decimal values from their calculator. Candidates should be aware what is expected of them in these types of problems.

90639 Sketch graphs of conic sections and write equations related to conic sections

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically

- could recognise graph shape from an equation and draw it accurately
- drew enough of a graph to clearly show all important features
- could write an accurate equation from a given graph
- understood the effect of transformations on a conic section
- substituted values to determine constants required when writing equations
- drew graphs given equations in standard or parametric form
- showed understanding of the meaning of asymptote and showed this on their graph
- used the “a” and “b” values in the equations given to locate the critical points on the graphs.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- drew graphs of poor quality
- did not draw enough of the graph being attempted to show understanding of its shape
- could not represent a graph transformation as a correct equation
- did not understand the parametric form given - some thought it represented a trig function
- were careless with the signs, powers, brackets and values of the variables in the equations of graphs

- could not transform equations algebraically, nor understand these transformations when sketching
- chose the incorrect x and y intercepts when sketching an ellipse, centred at the origin.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- could express the general form for a translated ellipse and follow a process to represent it fully
- understood implicit differentiation and could complete it correctly with the necessary algebra to represent the derivative correctly
- used the information given to write correct equations for the conic described in the question
- knew the relationship between the gradient of a tangent and its normal at a point
- could solve linear/non-linear simultaneous equations
- apply a model (conic equation) to a given context.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- understood the mathematics relating to tangent and intersection and could apply a process to answer the question
- demonstrated a logical chain of reasoning and had sufficient algebraic skills to carry out manipulations
- were confident with abstract, algebraic proof
- understood the need for rigour in a proof, and the need to demonstrate each step clearly
- showed determination to make multiple attempts before arriving at a solution
- were able to differentiate implicitly and substitute in a point to calculate a gradient
- were conversant with co-ordinate geometry techniques
- were confident with using indices, fractions and surds in their working.

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National Certificate of Educational Achievement

2012 Assessment Report

Statistics and Modelling Level 3

90642 Calculate confidence intervals for population parameters

90643 Solve straightforward problems involving probability

90644 Solve equations

90646 Use probability distribution models to solve straightforward problems

COMMENTARY

This was the final year for examinations to assess these achievement standards.

With the increased use of graphical calculators, candidates are able to obtain solutions and evaluate formulae by appropriately manipulating the technology. Achievement with Merit and Achievement with Excellence required understanding of concepts at Level 8 of the New Zealand curriculum for Statistics, and candidates needed to understand how to interpret and use the outputs from the graphical calculators. Candidates also needed to take more care with the presentation of answers; in particular, use of appropriate rounding linked to the context, the stating of rounding, the use of units where appropriate, and the use the correct notation.

STANDARD REPORTS

90642 Calculate confidence intervals for population parameters

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- calculated confidence intervals for means, difference-of-two-means or proportions correctly
- used the graphical calculator appropriately to obtain intervals and wrote these in an acceptable format
- used the normal distribution tables and formulae to obtain intervals and wrote these in an acceptable format.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- wrote confidence intervals in an unacceptable format
- inappropriately rounded answers
- incorrectly calculated the standard error or margin of error
- selected the incorrect type of confidence interval for the question
- were unable to identify population parameters.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- obtained a normal distribution z-score value for a given level of confidence
- rounded appropriately to obtain a minimum sample size
- formed and solved (in)equations correctly
- correctly interpreted a confidence interval to answer a question
- were aware that, in the absence of a given estimate for a proportion $\hat{p}$, it is appropriate to use an estimate of 0.5.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- calculated the mean and variance for the sum of independent random variables
- demonstrated an understanding of the theory behind confidence intervals
- communicated reasoning clearly and concisely
- completed problems involving an extended chain of reasoning with correct use of probability symbols and accepted notation.

90643 Solve straightforward problems involving probability

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- communicated the meaning of complementary events in context
- used a Venn diagram or contingency table correctly
- calculated the mean and standard deviation of a random variable.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- could not model a simple problem involving probability
- miscalculated the mean and/or standard deviation of a random variable
- multiplied two probabilities to find an intersection of events when the events were not independent
- used inappropriate diagrams or tables as part of their working
- confused complementary events with mutually exclusive events
- confused standard deviation and variance
- did not realise that probabilities should be between 0 and 1.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- calculated conditional probabilities
- demonstrated understanding of sampling without replacement and selected appropriate methods to solve related problems
- demonstrated skill with algebraic manipulation.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- selected and combined appropriate diagrams and methods to solve problems
- combined algebraic fractions to complete proofs

90644 Solve equations

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- optimised an objective function using linear programming by graphing, listing vertices and calculating associated profits
- performed two iterations of the bisection or Newton-Raphson method, showing the results of each iteration
- rearranged a simple equation correctly
- solved a 3 x 3 system of linear equations
- used their calculator correctly
- answered questions in context using clear correct statements.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- incorrectly graphed the linear inequalities
- misunderstood what two iterations of the bisection method meant
- used insufficient evidence when offering a solution
- demonstrated limited knowledge of the range of concepts assessed by this standard
- did not relate their answer to the problem.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- demonstrated knowledge of the range of concepts assessed by this standard
- wrote correct linear inequalities from information supplied
- recognised that an optimal solution is at the intersection of two of any of the constraints (including those vertices on the axes)
- answered the question posed in context
- understood what “two decimal places” meant
- realised that to solve using the Newton-Raphson method, two successive iterates must agree to required accuracy
- solved a 3 x 3 system of equations in context
- used units/variable names from the descriptions within the question.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- understood how to solve a linear programming problem when a vertex was not a whole number solution (and so not appropriate for the context)
- could explain how the Newton-Raphson method worked in detail
- explained why there a problem with the Newton-Raphson method in a particular instance

- understood the meaning of “inconsistent” and could apply this knowledge algebraically to find unknowns in a similar scenario.

90646 Use probability distribution models to solve straightforward problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- attempted only some of the questions
- demonstrated a basic knowledge of the normal and one other distribution
- solved problems involving a range of outcomes (cumulative)
- used graphics calculators or tables correctly.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- failed to select correct parameters
- found it difficult to deal with measurement units i.e. hours and minutes
- provided answers to one sig. fig. with minimal working
- did not show understanding of the meaning of “at least”
- demonstrated a basic knowledge of only the normal distribution.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- selected distributions appropriate to the question
- used correct parameters when calculating probabilities
- gave appropriate answers to the questions using appropriate rounding and units
- understood how to use the continuity correction for normal distribution
- combined two independent distributions.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- sustained extended reasoning
- took appropriate account of the detail given in the question
- performed inverse Poisson calculations
- found parameters for sums and differences of independent normal distributions.