National Certificate of Educational Achievement

2013 Assessment Report

Mathematics and Statistics

Calculus Level 3
Statistics Level 3

91577 Apply the algebra of complex numbers in solving problems

91578 Apply differentiation methods in solving problems

91579 Apply integration methods in solving problems

91584 Evaluate statistically based reports

91585 Apply probability concepts in solving problems

91586 Apply probability distributions in solving problems

CALCULUS

COMMENTARY

2013 brought new, but not radically different, standards and papers that featured a change in format from preceding years. The three papers consisted of three multiple part questions with each question giving the opportunity for candidates to receive a score out of eight. The sum of these scores gave a total for the paper. Allocated time for each paper was also increased.

This change in format gave candidates more opportunity to demonstrate their knowledge.

As a result the Merit and Excellence rates were up slightly in all papers relative to equivalent standards from the previous year.

Successful candidates displayed the ability to interpret a question, good algebraic skills and presented their answers in a clear, logical manner.

The ability to differentiate, or integrate, alone was not sufficient to pass the respective standards. Algebraic manipulation of expressions into forms appropriate for differentiating or integrating is vital. It is also essential in solving a problem once the differentiation or integration step has been carried out.

This year there were fewer candidates who attempted to answer questions using graphics calculators without first doing the required analytical steps.

STANDARD REPORTS

91577 Apply the algebra of complex numbers in solving problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- rationalised the denominator of a surd by correctly multiplying by the conjugate surd
- successfully manipulated complex numbers expressed in rectangular form or polar form
- represented a complex number on an Argand diagram
- applied the Conjugate Root theorem to find a second root for a given cubic equation
- substituted a complex number into a polynomial equation (using the factor theorem) and simplified the resulting equation
- applied the remainder theorem, or used long division, to calculate a remainder when a polynomial equation was divided by a linear expression
- successfully solved quadratic equations using either the quadratic formula or by completing the square, leaving their solution in simplified surd form
- recognised the symbol used for the conjugate of a complex number: $\bar{q}$
- found one solution for an equation of the form $z^n = a$.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- failed to manipulate complex numbers in either rectangular or polar form with accuracy
- misunderstood the Argand diagram
- could not apply the Conjugate Root Theorem
- could not apply the Remainder Theorem and/or the Factor Theorem
- failed to write the equation of a quadratic equation once the roots had been found
- manipulated surds incorrectly.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- solved a cubic equation with a pronumeral coefficient when given one of the roots
- manipulated a complex number in rectangular form and then calculated the exact form of its argument
- sketched a complex number on an Argand diagram to identify feasible arguments
- solved an equation involving the argument of the product of two complex numbers written in rectangular form with parameters
- wrote a quadratic equation when given information that allowed identification of its roots
- solved a proof involving manipulation of a complex number written in its general rectangular form and its conjugate
- simplified the quotient of two complex numbers expressed in rectangular form to a form with a rational denominator
- solved a surd equation involving a pronumeral
- solved an equation involving De Moivre's Theorem with the modulus expressed as a parameter
- re-wrote a surd equation in its equivalent quadratic equation.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- solved equations involving an argument statement
- knew to test all solutions of quadratic equations to determine if the solutions found were valid
- simplified and then manipulated a quotient of sums of complex numbers to a stage where they could separate the real and imaginary parts
- understood what was meant by "purely imaginary"
- recognised the equation of a circle and described it with its centre and radius
- manipulated a surd equation to its equivalent quadratic equation so that the discriminant properties could be applied.

OTHER COMMENTS

Students commonly used pencil to mark w on the Argand diagram which then meant that part would be ineligible for reconsideration.

91578 Apply differentiation methods in solving problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- correctly used the chain rule, product rule and the quotient rule
- were able to write the equation of a straight line using the gradient and one point
- understood how to find turning points
- differentiated trigonometric, logarithmic and exponential functions correctly
- had the ability to change an expression from surd form to exponent form
- correctly differentiated expressions with fraction exponents.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- made simple algebraic errors
- did not understand that $e^0 = 1$
- completed part of a question correctly but did not continue and answer the question
- confused integration and differentiation
- did not attempt the harder questions to gain some evidence of achievement.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- understood how to find the point of inflection
- could differentiate parametric equations and find gradients for parametric equations
- knew how to find stationary points
- understood the concepts of limits, continuity and differentiability.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- could convert the information given into a mathematical equation and use it to solve the given problem
- worked systematically and logically with clear reasoning
- used trigonometric identities to rewrite trigonometric functions so that they could be differentiated
- applied the chain rule correctly
- rearranged formulas to eliminate variables.

OTHER COMMENTS

Candidates must remember that they are communicating their answers to the examiner and so must present answers that are clearly written, readable and set out appropriately.

91579 Apply integration methods in solving problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- integrated functions
- before integrating, manipulated an expression into a suitable form if required
- used the trapezium rule to evaluate an integral of a function given values of the function
- calculated the area under a curve, including writing an integral for the area and/or showing the result of the integration
- integrated a function of the form $f'(x)/f(x)$.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- did not include the arbitrary constant with the result of an indefinite integral
- were unable, when necessary, to successfully manipulate an expression into a form so that it could be integrated
- integrated the product/quotient of two functions as the product/quotient of the integrals of the constituent functions
- took the value of n in the trapezium rule as the number of function values and not the number of intervals.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- applied integration to solve a problem involving kinematics (including correctly interpreting the meaning of "in the 4th second of motion" as "between $t = 3$ and $t = 4$" and not as "when $t = 4$" or as "between $t = 4$ and $t = 5$")
- applied integration to solve a problem given a rate of change
- understood the need, when integrating, to separate the variables if necessary
- solved differential equations of the form $\frac{dy}{dx} = ky$
- solved a problem involving an integral where the integrand was of the form $f'(x)/f(x)$.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- used integration to solve a complex problem involving areas
- solved problems involving differential equations of the form $\frac{dy}{dx} = ky$, including solving the differential equation
- worked systematically and logically
- showed their reasoning clearly
- used mathematical symbolism and terminology correctly.

OTHER COMMENTS

Brackets were very commonly omitted from expressions that required them.
Integral signs were very commonly omitted from expressions that required them.
The dx (or the equivalent) was very commonly omitted from integrals.
In Questions 1(d) and 3(e), a significant number of candidates did not solve the differential equation (as specified in the question) but simply wrote down the solution. Obtaining a correct answer in such cases was not accepted. Some candidates did not show a logical development of the solution (e.g. they omitted the constant when integrating) and this was not treated as a minor error. This examination paper requires candidates to show their ability to use integration methods, not their ability to remember the form of solution of a particular type of differential equation.
With definite integrals it was not uncommon for candidates to substitute the limits the wrong way round.

STATISTICS

COMMENTARY

Candidates are required to use knowledge up to and including Statistics Curriculum Level 8 when answering questions. Candidates are required to use the contextual information provided in the questions (or the reports) to inform and support their answers.

STANDARD REPORTS

91584 Evaluate statistically based reports

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- identified the target population clearly
- calculated a margin of error using an appropriate ‘rule of thumb’
- correctly calculated a confidence interval using percentages
- used a confidence interval to consider a “majority” claim
- identified explanatory and response variables
- demonstrated understanding of the basic concepts of an observational study and an experiment
- identified at least one potential difficulty in the wording of a selected survey question
- identified bivariate/correlation analysis using a linear regression model as an appropriate statistical process to describe the relationship between two variables.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- confused the target population with the targeted readers of the article
- confused the different ‘rule of thumbs’ for margin of error
- had no understanding of the terms explanatory and response variables
- misunderstood the term ‘non-sampling errors’
- commented on their own personal opinion, rather than on what was actually written in the report
- did not convert the margin of error to a percentage before using it to construct a confidence interval
- did not make the connection between the claims made in a report and the statistical process used to generate evidence or analysis for this claim
- did not understand the statistical meaning of ‘using the past as a source of data’ or ‘extending the results inappropriately’ or ‘confounding variables’
- misread the information presented or could not identify the significant statistical information given in the report.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- justified a claim using a confidence interval for the difference between two survey percentages
- demonstrated understanding of sampling variability and why the margin of error was included in statistical reports
- described non-sampling errors relevant to the survey information provided in the report
- understood the difference between sampling errors and non-sampling errors
- explained, with supporting evidence, why the sample would be representative
- made an appropriate inference for the target population from the calculated confidence interval
- understood the importance of sampling method and response rate in determining whether or not a sample was likely to be representative of the target population
- linked the claimed rate of change given in the statistical report to the gradient of a linear model
- linked to the context when giving a full explanation as to why the study was observational
- linked to the context when describing specific issues with causal relationship claims in a study
- recognised the need for the people to be randomly allocated into groups, and for one to be a control group, for an experiment
- recognised that the statistics to be compared were the means for the two groups, not proportions
- recognised the need for the people to be randomly allocated into two groups for an experiment
- critiqued a causal-relationship claim with justification using the available statistical evidence.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- identified specific and significant potential non-sampling errors based on the information available in the provided survey and explained how these could cause bias for that survey
- had a clear understanding of the meaning of bias and how this could lead to a sample that was not representative of the target population
- discussed the need to assess the strength of evidence for any inference
- acknowledged uncertainty as to the amount of sampling variation within the given population
- understood specific terms or phrases such as ‘confounding variables’, ‘extending the results inappropriately’ and ‘using the past as a source of data’.

OTHER COMMENTS

Candidates found it difficult to extract and use the information in the text of the reports and repeatedly made comments that were drawn from speculation. Candidates appeared to be following a set recipe for evaluating a statistical report, rather than using the information presented in the reports.

Candidates also demonstrated confusion between “sampling errors”, also referred to as sampling variability, and “non-sampling errors”, with the concept of sampling variability not well understood.

As surveys and polls are inevitably part of any statistically based report, candidates need to have an understanding of the basic principles in questionnaire design. Candidates also need to be familiar with basic statistical graphs and be able to suggest the use of appropriate statistical graphs to support claim.

91585 Apply probability concepts in solving problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- used probability trees appropriately to model a situation
- used two-way table appropriately to model a situation
- calculated and compared relative frequencies
- interpreted a bar chart showing frequencies of outcomes
- listed a set of outcomes satisfying a given condition
- used straightforward probability methods to solve problems.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- could not interpret the probability information provided in the text
- could not select an appropriate model for a given situation
- could not explain their ideas clearly in a given context
- confused relative frequencies and theoretical probabilities
- did not realise that probabilities greater than 100% indicated a problem.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- selected which probabilities were needed to solve a problem
- interpreted information from the question to calculate related probabilities
- used a Venn diagram to model a situation involving three events
- understood the difference between selecting with and without replacement
- used probability theory terminology to describe a situation in context
- decided when a problem required the use of conditional probability

- clearly described the elements of a situation involving probabilities.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- understood the difference between selection without replacement and independent events when solving a problem
- used set theory involving complementary events to put forward a convincing argument
- investigated a situation that involved chance and developed a generalisation
- applied basic algebra skills when generalising.

OTHER COMMENTS

Candidates often confused independent events with mutually exclusive events. All candidates should also be familiar with the notion of independent and dependent events, including conditional probability, and be prepared to use this knowledge across all questions.

Candidates need to be familiar with a range of probability tools and to use the information provided to select an appropriate one to use. They also need to link any information presented in tables, graphs or another visual form to the text provided, to avoid making assumptions about what the tables, graphs etc. represent. Candidates should be familiar with notions of risk, including relative risk, and associated vocabulary and methods such as “more likely” and baseline groups.

91586 Apply probability distributions in solving problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They typically:

- demonstrated understanding of a range of probability distributions, including the normal, Poisson, binomial and triangular distributions
- identified an appropriate probability distribution to use to solve a problem
- stated the probability distribution used, including its parameters
- calculated probabilities using an appropriate probability distribution (theoretical and experimental)
- described key features of a probability distribution shown in a graph (e.g. centre, spread, shape, unusual features, nature of random variable)
- calculated, or estimated, the mean of a probability distribution shown in a graph
- used descriptions such as “more than”, “at least” and “fewer” to correctly calculate the probability of cumulative outcomes
- used clear and correct probability statements as part of their answer.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They typically:

- confused the normal, Poisson, binomial and triangular distributions
- did not know what features of a probability distribution to identify
- failed to extract from the question the correct probability to be calculated
- misunderstood the meaning of the words “more than”, “at least” and “fewer”
- used incorrect probability methods e.g. multiplying two probabilities for an “OR” situation
- confused the mean and mode of a distribution
- mistakenly applied continuity corrections
- did not use common sense or graphs provided in checking their answers
- failed to do correct calculations using the triangular distribution formula from the formula sheet.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit typically:

- related the normal distribution features to that of a given histogram
- compared calculated normal distribution probabilities to that of a given histogram
- performed inverse calculations to find a parameter for a probability distribution
- justified the selection of probability distribution by linking conditions of the distribution with contextual information
- used probability methods to calculate combined events involving probability distributions
- considered chance as an explanation for the observed results, and attempted to use an appropriate probability distribution to investigate a claim
- used logical and sequential arguments when presenting their answers.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence typically:

- were able to consider the appropriateness of a probability distribution model by comparing an experimental distribution with a theoretical model, using significant features of the experimental distribution and relevant appropriate calculations to support their discussion
- fully explained assumptions used, in context, when applying probability distributions
- understood chance variation and could apply this sensibly, using an appropriate probability distribution, when investigating a claim
- clearly articulated each step of their answer, using a chain of logical reasoning.

OTHER COMMENTS

Candidates need to be aware that all probability distributions have a mean and standard deviation, not just the normal distribution. They also need to be familiar with describing the features of the probability distribution, including discussing the shape using appropriate and correct vocabulary, estimating the mean and standard deviation and identifying discrete and continuous random variables.

Candidates need to clearly identify the name of the distribution being used to solve a problem and state the parameters of this distribution. They should also write clear statements of what probability they are calculating at each step of their working; use correct probability statements, rather than details of what was entered into their calculator.

Candidates need to be aware that justifying the selection of a probability distribution model requires linking conditions of the model with the context, and restating these conditions in context.

Candidates may be required to make assumptions as part of solving a problem, and these need to be clearly stated or given when asked. Assumptions are needed when information is not known but is needed in order to be able to perform calculations or use a probability model.