National Certificate of Educational Achievement

2014 Assessment Report

Mathematics and Statistics

Calculus Level 3
Statistics Level 3

91577 Apply the algebra of complex numbers in solving problems

91578 Apply differentiation methods in solving problems

91579 Apply integration methods in solving problems

91584 Evaluate statistically based reports

91585 Apply probability concepts in solving problems

91586 Apply probability distributions in solving problems

CALCULUS

COMMENTARY

2014 was the second year of new standards. The papers were in a similar format to the previous year and gave a similar overall distribution of grades.

Successful candidates displayed the ability to read and interpret a question, reliable algebraic skills and presented their answers in a clear, logical manner.

Sound algebraic skills remain the cornerstone of this course.

Algebraic manipulation of expressions into forms appropriate for differentiating or integrating is vital. It is also essential in solving a problem once the differentiation or integration step has been carried out. Being able to differentiate, or integrate, is not on its own sufficient to pass the respective standards.

There remains a strong positive correlation between the standard of presentation of answers and the final grade for the paper.

This year there were fewer students who attempted to answer questions using graphics calculators without first displaying the required analytical steps.

STANDARD REPORTS

91577 Apply the algebra of complex numbers in solving problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They commonly:

- demonstrated familiarity with the factor theorem and competence with solving linear equations
- demonstrated the ability to convert a complex number from rectangular form into polar form
- used De Moivre's Theorem to find the fourth power of a complex number expressed in polar form
- understood the difference between solving a power equation and raising a complex number to a power
- found one of the four possible solutions of the equation z^4=-4k^2 i or found the correct arguments of all four of the solutions with the moduli incorrect
- substituted x+yi for z into the equation of the locus and correctly collected the real and imaginary parts
- solved a quadratic equation with complex roots and gave an answer in its fully simplified form
- applied the factor theorem to evaluate the unknown pronumeral coefficient of a cubic equation
- used the conjugate root theorem to find the unknown quadratic factor of a cubic equation

- sketched the appropriate circle needed to represent the locus of points satisfying the given equation
- rationalised the denominator of a complex number expressed as the quotient of two different complex numbers
- expanded and simplified three brackets involving surds
- correctly read complex numbers represented on an Argand diagram, and correctly plotted complex numbers on an Argand Diagram
- demonstrated an understanding of the relationship between the discriminant of a quadratic equation and the number of roots
- manipulated complex numbers in rectangular form, including powers, conjugates and reciprocals.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They commonly:

- attempted to use long division rather than the factor theorem to find the value of p
- made arithmetic errors when rearranging and solving equations
- incorrectly converted complex numbers from rectangular to polar form and vice-versa
- squared term by term when attempting to solve a surd equation
- failed to understand that they needed to substitute z=x+yi and worked with z when trying to find the equation of the locus
- inaccurately used the quadratic formula to solve an equation
- inaccurately simplified the roots that they did find for the quadratic equation
- could not identify the equation of a circle from a given locus equation
- could not sketch the equation of a circle
- incorrectly multiplied complex numbers in rectangular form when rationalising the denominator of a complex number
- were not able to multiply three brackets involving surds and simplify their result without careless errors
- failed to recognise that an equation involving the discriminant would be required to find out when a parabola would not have any x-axis intercepts.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit commonly:

- solved a surd equation to find the one valid solution
- demonstrated an understanding of the implications of squaring when solving an equation with surds
- solved an equation of the form z^n=a
- correctly wrote an expression for the modulus of each of the expressions involved in the equation of the locus
- solved a cubic equation involving a pronumeral coefficient either by using the factor theorem or by using the conjugate root theorem and then comparing coefficients
- sketched the graph of the circle described by the given locus equation and identified the maximum real value shown by a point on the circle

- rationalised the denominator of z and interpreted the statement arg (z) = π/4 to mean that im(z)/re(z) = tan π/4
- solved the appropriate inequation to find the values of p that would give parabolae without any x axis intercepts
- completed the four required proofs in question 3 (e) with the assumption that a=1
- completed three of the four required proofs by comparing the coefficients of two equal equations.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence commonly:

- found the equation of the locus described without making any careless errors
- demonstrated the ability to find the modulus of complex numbers expressed in rectangular form
- rationalised the denominator of z, interpreted arg (z) = π/4 and once the real and imaginary parts of z were made equal to each other, completed the proof
- found the equation with the given roots involving a coefficient for x^3 of a and then completed the proofs by comparing the coefficients
- found the equation with the given roots involving a coefficient for x^3 of 1 and also divided the given cubic equation through by a so that both of the equations were then equal and their coefficients could be compared.

OTHER COMMENTS

Question 2 (d) required candidates to draw the graph of a circle given the equation. It was clear that a large number of candidates did not know how to do this. Circles are no longer in the Level 2 Graphs standard, and many schools do not include the Conics topic in their Calculus course. Circles are a fundamental concept of Mathematics, however, and it is essential that a Level 3 Calculus candidate encounters the graph of a circle somewhere in their High School Mathematics career.

Question 3 (c) was done very poorly by the candidates. In many cases, it was left blank and in others candidates were either unsure how to use the discriminant to solve the problem or unable to solve a quadratic inequation. These types of questions are common at Level 2 and clearly these candidates were not prepared to see them at Level 3.

91578 Apply differentiation methods in solving problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They commonly:

- differentiated at least some of trigonometric, rational, exponential and logarithmic functions
- differentiated functions using the chain rule
- differentiated functions using the product and/or quotient rules
- found derivatives of functions and successfully used them to find gradients.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They commonly:

- made careless errors such as not copying a given function correctly or changing an expression in the middle of their working (e.g. 2x - 2 became 2x + 2)
- were unable to rearrange a function into a form suitable for differentiation
- displayed inadequate facility with fractional and negative exponents
- lacked algebraic skills such as expanding brackets
- failed to use brackets where they were required.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit commonly:

- correctly found an expression for the derivative of a function defined parametrically
- recognised the need to use the formula for sin2A when it was required
- made connections between different aspects of a problem, for example relating rates of change together or substituting for one of the variables in a function containing two variables before differentiating
- recognised several features of a function y = f(x) from its graph, including non-differentiability, the limit of the function as x approaches a given value, and the relationship between concavity and the second derivative of the function.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence commonly:

- solved problems requiring derivatives to be found and used in a chain of reasoning
- displayed well-developed algebraic skills
- worked carefully, systematically and logically
- expressed their reasoning clearly
- sketched a diagram to assist their reasoning when appropriate
- were prepared to start solving a problem again if a strategy proved non-productive or appeared to be getting unnecessarily complicated.

OTHER COMMENTS

Some candidates made their working unnecessarily complicated by working with expressions that could be simplified but that they did not simplify, e.g. in Question 3(e) they obtained 40/20 = (40 r)/h so h = (800 - 20r)/40 and then substituted (800 – 20r)/40 for h in the expression for the volume of the cylinder.

Some candidates made their working unnecessarily complicated by using the product or quotient rule when it was unnecessary, eg using the quotient rule to differentiate 32/x^2 in Question 3(b).

In Questions 1(e) and 3(e) a significant number of candidates simply substituted the numerical values given into expressions for the volume before differentiating. They then tried to differentiate a constant expression rather than an algebraic one.

91579 Apply integration methods in solving problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They commonly:

- correctly integrated using negative indices, exponential and trigonometric functions and surds
- recognised when to use the natural log to integrate
- correctly applied Simpson's Rule
- calculated definite integrals with limits in the correct order
- recognised the second derivative notation and integrated appropriately, correctly substituting for x, y and dy/dx.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They commonly:

- completed the definite integral in the incorrect order
- made mistakes with negatives
- substituted into formulas incorrectly
- could not apply Simpson's rule correctly or used the trapezium rule in error
- integrated all negative exponents as natural log
- could not integrate trigonometric expressions with constants in the expression
- confused integration and differentiation.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit commonly:

- demonstrated they could form and correctly use a model that involved a differential equation
- correctly substituted into an equation to find the value of a constant of integration
- correctly calculated definite integrals
- correctly separated variables to integrate
- successfully interpreted 'word problems' and substituted the correct values when required
- integrated trigonometric expressions and correctly used radians to find solutions
- integrated using surds.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence commonly:

- set out work logically and clearly, demonstrating clear logical steps to complete a proof
- showed an extended chain of reasoning to follow complex problems to their conclusion
- interpreted complex notation including parameters to integrate correctly
- correctly found the point of intersection for complex curves

- demonstrated they could use integration techniques in solving problems that required extended abstract thinking, and they could communicate mathematical insight.

OTHER COMMENTS

The correct evaluation and application of the constant of integration was beyond some candidates. Some candidates omitted this constant altogether. Others assumed that if an initial value of a function is zero then the constant of integration must also be zero, regardless of the nature of the function. Inability to deal correctly with the constant of integration stopped many candidates from getting any grade above Achieved, and also stopped some candidates from getting Achieved.

A few candidates used their graphic calculator to calculate values but needed to take note that it is necessary to show the correct result of integration before credit can be given. Clear setting out of solutions not only assists the markers to give credit for achievement, but enables candidates to follow clear and logical steps leading to solutions. Too many candidates started correctly but made mistakes because of poor setting out.

STATISTICS

COMMENTARY

Candidates are also required to use knowledge up to and including Statistics Curriculum Level 7 when answering questions.

Candidates are required to use the contextual information provided in the reports or questions to inform and support their answers.

Candidates need to show all necessary working and to take care to communicate their thinking clearly.

STANDARD REPORTS

91584 Evaluate statistically based reports

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They commonly:

- calculated a margin of error using an appropriate ‘rule of thumb’
- identified appropriate survey percentages with which to use the ‘rule of thumb’ margin of error (30%–70%)
- constructed confidence intervals using appropriate methods
- used a confidence interval to consider a “minority” claim
- identified whether a sample was likely to be representative (for the target population)
- identified an issue with a sampling method
- recognised the characteristics of observational studies and experiments
- described the explanatory and response variables for a study
- described at least one potential difficulty with the wording of a survey question or its response categories
- described and used features of a scatterplot when considering a claim
- identified causal claims within reports.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They commonly:

- did not take into account sampling variability when assessing claims based on survey percentages
- were unable to calculate margins of errors
- could not use margin of errors correctly to create confidence intervals
- misunderstood the term 'non-sampling errors’
- did not use the information provided in the report as the basis for their response
- confused the types of graphs commonly used in reports
- did not identify target populations as part of their responses.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit commonly:

- demonstrated familiarity with a range of sampling methods e.g. simple random, cluster, convenience
- used clear statistical language to communicate why the margin of error should be used when interpreting survey percentages
- explained that the 'rule of thumb' margin of error would over-estimate the actual margin of error for survey percentages less than 30% (or greater than 70%)
- could explain potential issues with survey questions with respect to the claims made in a report
- described relevant non-sampling errors
- described whether a sample was likely to be representative for the target population by considering the sampling method(s) or relevant selection biases
- used confidence intervals to make valid inferences about the target population(s)
- described and used features of graphs provided within reports when evaluating claims
- demonstrated a clear understanding of the difference between observational studies and experiments (by linking to the context provided) and the implications for any claims made based on these studies.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence commonly:

- demonstrated a very good understanding of the different methods needed to construct confidence intervals from survey percentages, in particular when to use 1.5 x average MOE and when to use 2 x MOE when comparing percentages
- constructed and interpreted a confidence interval for the difference between two population percentages (two independent groups) as part of their consideration of a claim
- identified specific and significant potential non-sampling errors based on the information available in the report and explained how these could cause bias
- understood specific terms or phrases such as 'confounding variables', 'extending the results inappropriately' and 'using the past as a source of data' and were able to provide examples of these and consider improvements for future studies
- demonstrated a clear understanding that a representative sample is needed to make generalisations or inferences for a target population and used specific examples to illustrate this using information presented within the reports
- demonstrated familiarity with a wide range of graphs and could use features of these graphs to assess claims made in reports, both in terms of assessing the strength of statistical evidence and determining potential non-sampling errors.

OTHER COMMENTS

Candidates often wrote general comments that were not related to the context or the questions being asked. Candidates need to extract and use the information in the text of the reports rather than make comments drawn from speculation. Many did not read or interpret the graphs provided within the reports and gave non-specific answers as a consequence.

Sampling variability was not well understood, with candidates commenting on the variability within a sample, rather than the variability of the sample estimates expected from repeated random samples.

Candidates needed to take more care with presenting any confidence intervals used as part of their answer, using appropriate notation and the percentage symbol, for example (23.5%, 48.5%)

Candidates should be prepared to use knowledge from across the statistical investigations achievement objectives of the Statistics curriculum up to and including Level 7. It is expected that candidates can describe features of a variety of graphs and link these to comments made within a report as part of their response.

91585 Apply probability concepts in solving problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They commonly:

- used two-way tables appropriately to model situations
- used probability trees appropriately to model situations
- calculated expected numbers
- calculated risks
- interpreted the word 'or' correctly for two events
- were familiar with different representations of random variables for single, combined and conditional events
- used straightforward probability methods to solve problems
- used clear and correct probability statements as part of their working
- interpreted probability information presented graphically.

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They commonly:

- could not interpret the probability information provided in the text
- could not select an appropriate probability tool or representation to solve a problem
- did not realise that probabilities greater than 100% indicated a problem
- confused frequencies with relative frequencies and expected numbers
- did not understand the difference between experimental and theoretical probabilities
- demonstrated a lack of familiarity with probability simulations and their results
- demonstrated a lack of understanding of chance variation
- could not explain their ideas clearly in a given context.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit commonly:

- compared risks using appropriate calculations and written statements
- understood the difference between selection without replacement and independent events
- selected and combined appropriate probability methods to solve problems
- demonstrated that two events were not independent using appropriate statistical reasoning
- expected to see a difference between a theoretical probability and an experimental probability.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence commonly:

- used a range of representations and tools when solving problems including but not limited to probability distribution tables and graphs, Venn diagrams, two-way tables and probability trees
- used reverse processes and probability formulae to solve problems where the probability information provided required manipulation
- clearly communicated the strategy used to solve a problem
- related answers to the context of the problem
- understood that chance variation is caused by random (selection) processes
- demonstrated familiarity with using the results of a simulation to explore the expected variability of an experimental probability.

OTHER COMMENTS

Many candidates struggled to demonstrate an understanding of randomness and chance variation.

Candidates should be familiar with the notion of independent and dependent events, including conditional probability, and be prepared to use this knowledge across all questions.

91586 Apply probability distributions in solving problems

ACHIEVEMENT

Candidates who were awarded Achievement for this standard demonstrated the required skills and knowledge. They commonly:

- calculated the mean of a discrete random variable from a probability distribution table
- demonstrated understanding of a range of probability distributions
- selected appropriate probability distribution models
- stated the probability distribution used, including its parameters
- calculated probabilities using a range of probability distributions
- used probability distributions presented in tables and graphs

- used probability methods to calculate the probability of combined events
- used clear and correct probability statements as part of their answer
- described key features of a probability distribution presented in a graph (e.g. centre, spread, shape, unusual features).

NOT ACHIEVED

Candidates who were assessed as Not Achieved for this standard lacked some or all of the skills and knowledge required for the award of Achievement. They commonly:

- confused the normal, Poisson, binomial, triangular and uniform distributions
- demonstrated unfamiliarity with probability distribution graphs
- failed to extract from the question the correct probability to be calculated
- did not state their answers in context
- presented working that was unclear or insufficient to support their answer
- failed to give explanations of their calculations
- used incorrect probability methods
- applied continuity corrections when these were not appropriate.

ACHIEVEMENT WITH MERIT

In addition to the skills and knowledge required for the award of Achievement, candidates who were awarded Achievement with Merit commonly:

- justified the selection of probability distribution by linking conditions of the distribution with contextual information
- clearly stated any assumptions made in applying probability distributions
- performed inverse calculations to find a parameter for a probability distribution
- described any assumptions of independence in context
- identified and described why an assumption may be invalid for a given probability distribution and context
- compared key features of an experimental distribution and a theoretical distribution visually using a graph
- used logical and sequential steps when presenting their answers.

ACHIEVEMENT WITH EXCELLENCE

In addition to the skills and knowledge required for the award of Achievement with Merit, candidates who were awarded Achievement with Excellence commonly:

- were able to consider the appropriateness of a probability distribution model by comparing an experimental distribution with a theoretical model, using significant features of the experimental distribution and relevant calculations to support their discussion
- demonstrated a range of knowledge and methods when applying probability distributions to solve problems, for example but not limited to, the use of formulae, graphics calculators, similar triangles, probability distribution tables and graphs
- discussed at least one limitation (or weakness) of applying a specific probability distribution
- clearly communicated the strategy used to solve a problem
- related answers to the context of the problem.

OTHER COMMENTS

Candidates needed to be aware of the importance of linking their answer to the context of the question. This may apply to stating conditions for a particular distribution, where the generic conditions need to be linked to the context. This may also apply to the selection of a particular distribution for a given situation where the choice of model needs to be justified with reference to the context.

Candidates needed to understand that a probability distribution is a model for a real life situation and appreciate that it will not be a perfect fit. They may also be required to discuss potential limitations of using that distribution with reference to the context.

Candidates needed to be aware that all probability distributions have a mean and standard deviation, not just the normal distribution. They also needed to be familiar with describing the features of the probability distribution, including discussing the shape using appropriate and correct vocabulary, estimating the mean and standard deviation and identifying discrete and continuous random variables.

Candidates needed to clearly identify the name of the distribution being used to solve a problem and state the parameters of this distribution. They should also write clear statements of what probability they are calculating at each step of their working, use correct probability statements, rather than details of what was entered into their calculator.

Candidates may be required to make assumptions as part of solving a problem, and these needed to be clearly stated or given when asked. Assumptions are needed when information is not known but is needed in order to be able to perform calculations or use a probability model.