2015 NCEA Assessment Report

Calculus Level 3 91577, 91578, 91579
Statistics Level 3 91584, 91585, 91586

Part A: Commentary

The three 2015 papers were in a similar format to the previous year and gave a similar overall distribution of grades.
The fundamental skills required for success in these papers remain the same.
Firstly: Sound algebra skills. These are necessary to manipulate expressions into the form required for integrating or differentiating, and to solve a problem once the integration or differentiation has taken place.
Secondly: Concise, logical setting out of working. A neat, clear presentation of work does not guarantee the correct solution, but a haphazard, jumbled presentation almost certainly guarantees an incorrect answer.
Thirdly: The ability to reliably differentiate and integrate the standard functions associated with the standards. This often relies on the candidate's ability to manipulate a function into the form required, see above.
A point noted by the panel leaders was that some candidates continued working through answers that became increasingly difficult and more complicated. Sometimes these longer alternative methods lead to the desired result, but more often they led nowhere and wasted a considerable amount of valuable time. There should be a point where candidates realise that they have embarked down the wrong track. The best option at this point is to go back to the question and check the setup of their solution, initial working or a possible missed simplification in the early stages of their answer.

Part B: Report on standards

1. Assessment Report for 91577: Apply the algebra of complex numbers in solving problems

Achieved	Candidates who were assessed as Achieved commonly: • solved an equation by applying the quadratic formula or completing the square • simplified an expression involving surds • manipulated complex numbers both in rectangular form and polar form • successfully represented a complex number on an Argand diagram • solved problems involving the use of the discriminant • solved problems involving the use of the Remainder Theorem and the Factor Theorem • grouped real and imaginary parts of a complex number • calculated the argument of a product of complex numbers correctly.
Not Achieved	Candidates who were assessed as Not Achieved commonly: • could not simplify a surd • could not 'complete the square' of a simple quadratic • could not use an algebraic method to solve a quadratic equation • failed to accurately manipulate complex numbers in either rectangular or polar form • failed to represent a complex number accurately on the Argand diagram • misunderstood how to use the Remainder Theorem for calculating a remainder • did not know how to calculate the argument of a complex number expressed in rectangular form • could not apply their knowledge about the discriminant to solving problems involving different types of roots.
Achieved with Merit	Candidates who were assessed as Achieved with Merit commonly: • successfully equated the real and imaginary parts of two complex numbers in order to find unknown coefficients • understood that the discriminant needed to be rewritten in perfect square form to prove that roots of the quadratic equation provided would always be real • successfully simplified a quotient of complex numbers by multiplying both the numerator and the denominator by the conjugate of the denominator

	• manipulated rational expressions and equated coefficients • manipulated powers of i when simplifying a quotient involving complex numbers • solved a cubic equation with a pronumeral coefficient with one root provided • solved an equation of the form $z^n = x + yi$ involving pronumerals by converting from rectangular to polar form and applying De Moivre's Theorem • found the roots of a simple polynomial equation of degree 5.
Achieved with Excellence	Candidates who were assessed as Achieved with Excellence commonly: • communicated their thinking clearly about what they were doing while completing multi-step problems • completed the required proof by making connections between factors and roots and then manipulating the resulting equations involving pronumerals • simplified the quotient of two complex numbers and understood that they would need to equate the real and imaginary parts since the argument was pi/4 in order to find the required Cartesian equation • found complex roots and applied de Moivre's Theorem with clear communication of the process.
Standard specific comments	Knowledge and use of De Moivre's theorem is fundamental to Complex Numbers. Candidates need to be able to calculate the modulus and argument when converting a complex number from rectangular to polar form. Many candidates found difficulty in calculating the argument of a complex number. The difficulty often stemmed from confusion as to which quadrant the complex number was in. The presence of the pronumeral 'k' seemed to cause difficulty for a number of candidates in question 3d. Some candidates misinterpreted the conjugate root theorem. In question 3c, it was common for these candidates to indicate their belief that if the equation provided had a solution of x = -2 then the equation would also have to have a solution of x = 2. The setting out of proofs was generally done poorly. In particular the concept of using a perfect square to show that an expression must always be positive, as in question 1d, was not well understood.

2. Assessment Report for 91578: Apply differentiation methods in solving problems

Achieved	Candidates who were assessed as Achieved commonly: • differentiated trigonometric, rational, exponential and logarithmic functions successfully • differentiated functions using the chain rule successfully • differentiated functions using the product and/or quotient rules successfully • found derivatives and used them correctly in relation to gradients of tangents and normal • demonstrated sound algebraic manipulation skills.
Not Achieved	Candidates who were assessed as Not Achieved commonly: • were unable to rearrange a given function into a form suitable for differentiation • used the product or quotient rule unnecessarily • differentiated constants as if they were variables • substituted the given value of a variable into expression(s) before differentiating in problems involving rates of change • lacked ability in working with fractional and negative exponents • displayed inadequate algebraic skills - for example when expanding brackets • did not use brackets when required.
Achieved with Merit	Candidates who were assessed as Achieved with Merit commonly: • understood the conditions for an increasing function and could express the required regions correctly using inequalities • were able to find the expression for the derivative of a function defined parametrically • recognised features of continuity, limit, differentiability from a graph • could show the process of differentiating a given expression for x to equal a given differential equation • understood how initial, origin and velocity information for a particle could be used to set up and solve equations.

Achieved with Excellence	Candidates who were assessed as Achieved with Excellence commonly: • took time to use the information provided to set up the problem with correct calculus/algebraic statements and appropriate related rates of change • answered a problem systematically with clear, logical reasoning • used trigonometric relationships and identities to set up an expression that could be differentiated • correctly rearranged formulae to eliminate unnecessary variables.
Standard specific comments	Candidates must use the proper notations when writing derivatives and understand the need to rearrange expressions to a single variable before differentiating. Having rearranged the parametric equations correctly in Q3(c) some candidates calculated the trigonometric relations for fractions of phi in degrees mode on their calculator. In Question 1(e) candidates did not realise that some terms in the expression they set up were constants and could not be differentiated. Expressions for sin 30, cos 30 or tan 30 were often differentiated to cos 30, etc. Unnecessary algebraic expansions took a lot of time for the candidate to complete and were usually incorrect. Candidates had either forgotten or did not know the appropriate rule that could have simplified their working. This happened quite often with Q1(b).

3. Assessment Report for 91579: Apply integration methods in solving problems

Achieved	Candidates who were assessed as Achieved commonly: • integrated polynomial, exponential and trigonometric functions correctly • correctly rearranged expressions into a form which allowed successful integration • successfully found the constant of integration given x and y values • successfully applied the trapezium rule.
Not Achieved	Candidates who were assessed as Not Achieved commonly: • incorrectly integrated trig functions, in particular getting the sign wrong when integrating sine or cosine • incorrectly integrated any negative power of x, or any function with a denominator which contained a function of x, to a logarithm • incorrectly rearranged formulae before integration • misinterpreted the number of intervals as the number of x-values when using the trapezium rule • misunderstood the 'signed area' concept associated with definite integration.
Achieved with Merit	Candidates who were assessed as Achieved with Merit commonly: • manipulated formulae into a form suitable for integrating successfully • expressed (Ax+B)/(Cx+D) = E +F/(Cx+D) correctly • manipulated surds correctly • wrote and solved a differential equation of the form dP/dt=kP • found the intersections of two functions correctly • found the area between the graphs of two functions correctly.
Achieved with Excellence	Candidates who were assessed as Achieved with Excellence commonly: • followed an extended chain of reasoning/algebra to a successful conclusion • manipulated a trigonometric function into a form that was suitable for integration • successfully interpreted and used an unfamiliar formula involving integration • recognised that (e^kt)^2 = e^2kt • demonstrated sound algebraic skills.
Standard specific comments	Question 3b was the most poorly completed question in the paper. It tested a simple understanding of the difference between an area under a curve and a definite integral. The great majority of candidates did not understand the concept of 'signed area'. When solving the differential equation in question 1c many candidates incorrectly assumed that y = 4 when x = 0 equates to the constant of integration being 4. This type of assumption, regardless of the nature of the function being integrated, is a common error that has been noted in assessment reports for a number of years.

2015 NCEA Assessment Report

Statistics Level 3 91584, 91585, 91586

Part A: Commentary

Comment on the overall response of candidates to 2015 examinations for all achievement standards covered by this report.

Candidates who performed well across the standards were able to use the contextual information provided in the statistical reports or questions to inform and support their answers. This contextual information was integrated with statistical understandings, such that the responses were not generic, opinion-based or speculative in nature.

Successful candidates were also able to extract the required information from the question and use this to select and combine appropriate methods or understandings as part of their response. In providing responses to questions, these candidates showed all necessary working and take care to communicate their thinking and reasoning.

Some candidates demonstrated a lack of understanding of concepts which are introduced at lower levels of the Statistics curriculum. Examples include but are not limited to: using simulations to estimate probabilities, using appropriate language for describing the shape of distributions, and understanding the relationships between data, probability and statistical inference.

While each of these standards assesses different curriculum level eight achievement objectives, candidates needed to be more aware of inter-connectedness of these achievement objectives. For example, all standards required students to work with data. This may have been from the perspective of making an inference about a population parameter, in order to estimate the probability of an event or a risk, or as part of selecting or considering a probability distribution model.

Part B: Report on standards

1. Assessment Report for 91584: Evaluate statistically based reports

Achieved	Candidates who were assessed as Achieved commonly: • understood the terms explanatory and response variable • distinguished between different study designs, specifically observational studies and experiments • identified potential issues with a statistical process or claim presented in a statistical report • understood the need for a representative sample of the target population • calculated a margin of error using an appropriate 'rule of thumb' • understood the concept of error bars in bar charts • demonstrated an understanding of the relationship between the margin of error, the confidence interval, and the sample size • were familiar with different sampling methods • used a margin of error correctly to construct a confidence interval.
Not Achieved	Candidates who were assessed as Not Achieved commonly: • did not read the statistical report carefully before answering questions • lacked understanding about the concepts involved with observational studies and experiments • were not able to show an understanding of sampling methods, sampling error and non-sampling errors • confused the margin of error with a confidence interval • could not identify and use the sample size for a margin of error • could not answer in context but instead provide definitions (such as cluster sampling) • presented hearsay or speculation rather than information extracted from the

	provided statistical reports • did not understand that representativeness is not necessarily linked to sample size, e.g. 1500 people can represent a large population, if sampled correctly • focused on one or two of the questions rather than attempting all three questions.
Achieved with Merit	Candidates who were assessed as Achieved with Merit commonly: • described potential issues in a statistical report in terms of how they relate or affect the statistical information or claims provided in the report, including issues with the sampling method used, the questions asked, or the setting and method used for the data collection • demonstrated understanding of the relationship between the study design and the limitations of the inference(s) made, for example, that causal claims, in general, cannot be made from observational studies • understood and described why weighting is used in a sampling context • demonstrated an understanding of the different sampling situations and the associated margin of errors • linked the size of the margin of error to the length of the error bars • understood that the margin of error would be smaller with survey proportions below 30% or above 70%, and so the 'rule of thumb' margin of error over-estimates the margin of error in these cases • could re-arrange the rule of thumb formula to calculate the sample size.
Achieved with Excellence	Candidates who were assessed as Achieved with Excellence commonly: • analysed statistical information presented in a report to decide if the evidence is sufficient or adequate for claims being made in the statistical report • applied knowledge of confounding variables to discuss how these could interact with both variables in a study • applied knowledge of non-sampling errors by identifying, describing and critically evaluating a specific feature of the statistical report • constructed and interpreted appropriate confidence intervals in response to a claim made based on the difference between two proportions • were succinct in their responses and presented clear well-supported discussion points rather than lengthy generic statements or descriptions • integrated statistical and contextual knowledge in their responses when discussing issues with a statistical process or claim presented in the statistical report.
Standard specific comments	Candidates often wrote general (generic) statements that were not related to the context of the question being asked and appeared to be rote learned. Candidates needed to use specific evidence from the statistical report in their answers. Specifically, when discussing non-sampling errors, candidates needed to identify the potential issue (contextual understanding), explain why this is an issue (statistical understanding), and discuss the implication of this issue for the claim(s) being made in the statistical report (integrating contextual and statistical understanding). Understanding of sampling variability was not well demonstrated by candidates, with candidates commenting on the variability within a sample, rather than the variability of sample estimate(s). At this level, by quantifying the margin of error using the 1/root(n) rule of thumb, candidates should focus on the size of the sample and its effect on the expected variation of sample estimates. Candidates needed to take more care with presenting any confidence intervals used as part of their answer, and should use appropriate notation and the percentage symbol, for example (55.8%, 62.2%). Candidates also needed to explain what the confidence interval means and its implication back to the target population (the inference). Candidates were not prepared to use knowledge from across the statistical investigations achievement objectives of the Statistics curriculum up to and including Level 7. For example, it was expected that candidates are familiar with a range of sampling methods. Many candidates confused cluster sampling with stratified sampling.

2. Assessment Report for 91585: Apply probability concepts in solving problems

Achieved	Candidates who were assessed as Achieved commonly: • assumed independence of events in order to calculate the probability of a combined event • used contextual aspects of the chance situation being modelled in their answers • used two-way tables appropriately to model situations • used probability trees appropriately to model situations • calculated and compared risks • used straightforward probability methods to solve problems • used clear and correct probability statements as part of their working.
Not Achieved	Candidates who were assessed as Not Achieved commonly: • could not interpret the probability information provided in the text • could not select an appropriate probability tool or representation to solve a problem • could not calculate a risk or interpret a risk in context • did not understand the difference between experimental probabilities (data-based estimates) and theoretical probabilities (model-based estimates) • demonstrated a lack of familiarity with probability simulations and their results • demonstrated a lack of understanding of chance variation • could not explain their ideas clearly in a given context • focused on one or two of the questions rather than attempting all three.
Achieved with Merit	Candidates who were assessed as Achieved with Merit commonly: • selected and combined appropriate probability methods to solve problems • related probability concepts to the context of the chance situation being modelled • justified the assumption of independence to calculate a combined/joint probability • demonstrated that two events were not mutually exclusive using appropriate statistical reasoning.
Achieved with Excellence	Candidates who were assessed as Achieved with Excellence commonly: • used partitioning and other effective strategies for solving problems involving two or more events • developed a good chain of reasoning based on the properties of the probabilities of combined and conditional events • demonstrated familiarity with using the results of a simulation to assess whether there is evidence against chance acting alone • clearly communicated the strategy used to solve a problem • integrated contextual and statistical knowledge.
Standard specific comments	Candidates struggled to demonstrate an understanding of the true probability, and different ways to obtain estimates of this true probability using theoretical models or data (experimental probability). Many candidates appeared to be unfamiliar with the use of simulation to generate outcomes from a theoretical model, to assess the strength of evidence against chance acting alone. Many candidates also did not demonstrate they understood the difference between a theoretical (model) estimate and a data-based estimate (experimental probability).

3. Assessment Report for 91586: Apply probability distributions in solving problems

Achieved	Candidates who were assessed as Achieved commonly: • calculated the mean of a discrete random variable from a probability distribution table • selected appropriate probability distribution models • stated the probability distribution used, including its parameters • calculated probabilities for different probability distributions • used clear and correct probability statements as part of their answer • described key features of a probability distribution presented in a graph (e.g. centre, spread, shape, unusual features) • sketched a probability density function, such as the uniform distribution, with correct scales • demonstrated understanding that for a probability density function the total area under the density function is one.

Not Achieved	Candidates who were assessed as Not Achieved commonly: • confused the normal, Poisson, binomial, triangular and uniform distributions • demonstrated unfamiliarity with sketching probability density functions • lacked knowledge of the features and shape of distributions • did not state the conditions for probability distributions correctly or in context • failed to extract from the question the correct probability to be calculated • did not state their answers in context • presented working that was unclear or insufficient to support their answer • failed to give explanations of their calculations • used incorrect probability methods • misinterpreted ideas such as “at least”, “no more than”, etc. • focused on one or two of the questions rather than attempting all questions.
Achieved with Merit	Candidates who were assessed as Achieved with Merit commonly: • recognised the need to use methods associated with combined/joint and conditional probabilities • correctly stated the conditions for probability distributions, in context • justified the selection of probability distribution by linking conditions of the distribution with contextual information • clearly stated any assumptions made in applying probability distributions • performed inverse calculations or used their graphics calculator efficiently to find a parameter for a probability distribution • described any assumptions of independence in context • compared key features of an experimental distribution and a theoretical distribution visually using a graph • used logical and sequential steps when presenting their answers.
Achieved with Excellence	Candidates who were assessed as Achieved with Excellence commonly: • demonstrated confidence with the features and conditions of a range of probability distributions and could apply these to a variety of contexts • considered the appropriateness of a probability distribution model by comparing an experimental distribution with a theoretical model, using significant features of the experimental distribution, relevant calculations, and conditions of the probability distribution model, to support their discussion • demonstrated a range of knowledge and methods when applying probability distributions to solve problems, including but not limited to combining different probability distributions and using probability methods related to conditional events • clearly communicated the strategy used to solve a problem and related answers to the context of the problem.
Standard specific comments	Candidates needed to demonstrate an understanding that a probability distribution is a model for a real life chance situation. Candidates needed to be aware that when sample data is used to build or validate a probability distribution model, that they are engaging with statistical inference, and so associated concepts and understandings, such as the consideration of sample size, are relevant to any discussion about the model. Candidates needed to be aware of the importance of linking their answer to the context of the question. This may apply to stating conditions for a particular distribution, where the generic conditions need to be linked to the context. This may also apply to the selection of a particular distribution for a given situation where the choice of model needs to be justified with reference to the context. Candidates needed to clearly identify the name of the distribution being used to solve a problem and state the parameters of this distribution. They should have also written clear statements of what probability they are calculating at each step of their working, using correct probability statements rather than details of what was entered into their calculator. Candidates were required to make assumptions as part of solving a problem, and these needed to have been clearly stated or given when asked. Assumptions are needed when information is not known but is needed in order to be able to perform probability calculations or use a probability distribution model.