Assessment Report

On this page

Level 3 Calculus 2021

Standards 91577 91578 91579

Part A: Commentary

The 2021 papers followed a similar format to recent years' papers. Overall candidate performance, however, was also of a similar standard. Many candidates displayed the ability to solve problems in both conventional and innovative manners. There were also candidates who made fundamental errors which showed lack of understanding or carelessness.

Common points made by panel leaders were:

- Setting out of working in a clear, logical manner is a real advantage when trying to solve problems, in particular problems with which require an extended chain of reasoning.
- Failure to read the question carefully to clearly identify what needed to be calculated or what needed to be shown was an issue. For example, many candidates provided only the *x* coordinate when they were asked to find the coordinates of a point or provided only the gradient of normal when they w asked to give the equation of the normal.

- Candidates attempted to gain an Excellence grade by only attempting the questions that they interpreted to be ‘excellence’ questions. This strategy should not be encouraged as there is zero room for error. A large proportion of candidates who adopted this approach ended up with very poor results.
- Candidates who Not Achieved answered only one or two parts of each question. There are many opportunities to gain credit for correct working in problems that candidates may struggle to solve completely. Answering only a small portion of the paper does maximise a candidate’s chance of success.

Part B: Report on standards

91577: Apply the algebra of complex numbers in solving problems

Examinations

This paper gave candidates multiple opportunities to display their understanding of the material outlined in the standard. Namely:

- quadratic and cubic equations with complex roots
- Argand diagrams
- polar and rectangular forms
- manipulation of surds
- manipulation of complex numbers
- loci
- De Moivre’s theorem
- equations of the form $z^n = r \operatorname{cis}\Theta$, or $z^n = a + b\,\mathrm{i}$ where a, b are real and n is a positive integer.

A study of previous year’s examination papers and schedules will show that there are many skills which are regularly assessed. Questions requiring the application of De Moivre’s theorem occur regularly in this standard, so candidates should be well prepared for this type of problem, for example.

The requirement to show an answer in terms of a specific variable or in in a specific form is not understood or ignored by some candidates. Giving a complex number in the required form, rectangular or polar, for example.

Many candidates lacked the ability to form the expression needed to solve the problem that resulted or understand the algebra required to make progress toward an answer.

Candidates should realise that an explanation at the end of a calculation or a short concluding statement may be required to finish an answer.

This standard does not cover as much content as the other two external papers. As a result, the paper is more predictable than the other two and an organised revision programme which paid particular attention to previous papers would pay dividends.

Observations

There are fundamental skills that candidates who want to achieve success in this standard must have.

The ability to multiply and divide complex numbers in both rectangular and polar form is the most fundamental aspect of this standard.

The need for multiplying by a conjugate fraction was required several times in this exam. It should be a skill that is well practised by candidates in its various forms.

The skill of understanding the modulus of a complex expression was required several times in the paper.

Application of De Moivre's theorem is another cornerstone technique. Candidates need to be more careful with their answers for questions requiring the use of De Moivre's theorem when finding solutions to an equation. Candidates need to ensure they calculate the correct argument when converting the original equation to polar form. If this initial angle is incorrect then so is all the following work. This type of question needs to be given to setting up a general solution with a correct initial angle. Candidates had trouble representing the modulus correctly, so all other work done in the question was of no use.

Grade awarding

Candidates who were awarded **Achievement** commonly:

- solved an equation by using completing the square method or the quadratic formula
- manipulated complex numbers in either polar or rectangular form

- showed an understanding of what the argument and modulus of a complex number were
- rationalised a denominator correctly
- identified real and imaginary terms in an expression, and could group them correctly
- solved quadratics with unknowns as constants (either factorising or completing the square)
- understood the difference between, and used, the factor theorem and remainder theorem.

Candidates whose work was assessed as Not Achieved commonly:

- did not demonstrate basic algebra skills needed to solve, simplify, expand, factorise
- displayed little understanding of the process for completing the square
- did not rationalise a denominator correctly
- did not find the modulus of a complex number
- did not manipulate complex numbers in either polar or rectangular form
- converted a complex number from one form to another (rectangular to polar or vice versa) before performing calculations with complex numbers
- did not demonstrate converting to polar form, and/or understand what an argument is
- did not understand the remainder theorem.

Candidates who were awarded **Achievement with Merit** commonly:

- differentiated between the factors and solutions of an equation
- displayed a clear understanding of the remainder theorem and factor theorem
- understood the meaning of modulus and argument, and were able to express statements using these features correctly
- expanded quadratics involving surds, or imaginary numbers
- understood how to use De Moivre’s theorem, and could apply it correctly

- understood the meaning of “purely real” or purely imaginary” complex numbers, and could form and solve the equations that resulted
- substituted $z = x + iy$ into equations, and manipulated to separate them into real and imaginary parts
- identified that a complex number with equal positive real and imaginary parts has an argument of $\pi/4$.

Candidates who were awarded **Achievement with Excellence** commonly:

- used their algebra skills to accurately set up and solve equations without unnecessary or confusing statements in their working
- demonstrated problem solving skills required to group real and imaginary terms, and could apply the correct algebra to them
- understood what the modulus symbol required, often using their knowledge of loci
- showed understanding of the modulus being a length, and the implications for this in solving complex problems
- explored problems, recognising that square roots generate both a positive and negative solutions, but that not all solutions will be valid, and solved accordingly
- completed the required proof by making connections between real and imaginary parts, and completing the square
- communicated their thinking clearly and accurately about what they were doing while completing multi-step problems
- provided clear, logical, and easy to follow working out, reducing the chance of silly numerical or algebraic errors.

91578: Apply differentiation methods in solving problems

Examinations

This paper gave candidates multiple opportunities to display their understanding of the material outlined in the standard. Namely:

- derivatives of power, exponential, and logarithmic (base e only) functions

- derivatives of trigonometric (including reciprocal) functions
- optimisation
- equations of normals
- maxima and minima and points of inflection
- related rates of change
- derivatives of parametric functions
- chain, product, and quotient rules
- properties of graphs (limits, differentiability, continuity, concavity).

The first bullet point was a weakness for some candidates. More practice in differentiating the range of functions listed is needed, as is the application of the chain, product and quotient rules. The chain rule in particular was a source of numerous careless errors. An example of this is in Question 1(a) where the derivative of sin2*x* lost the 2 and/or the derivative of $3e^{3x}$ lost the 3. The denominator of the quotient rule also caused issues.

The need to solve problems requires candidates to have sound algebra skills, which come from continual practice. Many candidates did not know how to solve a simple quadratic equation, for example Q2b $x^2 + 2x = 0$, and it was common to see incorrect simplifying of factors in rational expressions. Candidates need more practice at solving equations involving exponential, logarithmic and trigonometric functions. Often they could find the derivative required, but lacked the algebra skills to use it to solve the problem.

Modelling for optimisation problems continues to be a skill that only the best candidates can handle – teachers need to find ways to find these types of problems accessible to a wider variety of candidates. The optimisation problem with multiple variables and constants (Q3e) was too hard for all but the very top candidates. Candidates need a lot more practice at understanding how to deal with multiple variables and constants within an optimisation problem.

Many candidates did not have a good understanding of parametric equations and were satisfied that substituting t = 10 at the point (10,0) would allow them to find the required gradient. More understanding of functions described parametrically would be useful for candidates.

Few candidates knew how to solve a problem involving a tangent through a point not on the given curve. This type of problem needs to be included in all teaching programmes. It was clear by the number of candidates who differentiated and substituted (–2,1) into the gradient function to find the equation of the tangent that they were not experienced in this type of problem-solving.

Observations

Good algebraic skills are crucial for success in this standard; expanding, factorising, simplifying expressions, manipulating algebraic fractions, and solving various equations, especially surd equations and quadratic equations but also equations involving exponential, logarithmic and trigonometric functions.

The correct use of brackets and accurate mathematical statements remains a challenge for many Calculus candidates. The ability to avoid making careless errors or having a good checking system to find and fix the careless mistakes is also important for this standard.

Failure to read the questions carefully meant that some candidates stopped before they had finished answering the question. For example, few candidates gave both coordinates for the point Q as required by the instructions of Q3(b). Also, few candidates provided the full proof by testing the maximum using the derivatives as required by the instructions of Q1(e).

Few candidates could form a model with one variable, as they fail to identify the variables and the constants in the scenario of Q3(e). Indeed, many candidates attempted to differentiate the function as it was written with multiple variables.

The concept of "gradient at a point" on a curve needs to be developed more with the candidates. For question 2(e), a lot of the candidates substituted the given *x*-coordinate directly into the gradient equation without realizing that the *x*-coordinate would not give the required gradient, since it is not on the given function.

Grade awarding

Candidates who were awarded **Achievement** commonly:

- used the chain rule, product rule and quotient rule correctly in combination with power functions, trigonometric functions, exponential and logarithmic functions
- solved quadratic equations
- provided the *x* coordinate correctly for any stationary points and surd

- provided the y coordinate correctly using the *x* coordinate
- recognised features of gradient, concavity, and limit from a graph
- used appropriate interval notation to describe when a piecewise function had zero gradient, or was concave up
- understood that the limit of a function could exist where there was a hole.

Candidates whose work was assessed as **Not Achieved** commonly:

- applied the chain rule, product rule or quotient rule in combination with power, trigonometric, exponential, and logarithmic functions
- did not solve equations algebraically, even though they had found the correct derivative
- did not correctly factorise and solve quadratic expressions
- did not demonstrate the understanding that there are 2 solutions when solving quadratic equations
- demonstrated poor algebraic skills such as in accurate expanding, factorising, and cancelling of factors.
- did not recognise features of gradient, concavity, and limit from a graph
- did not use inequality symbols correctly.

Candidates who were awarded **Achievement with Merit** commonly:

- solved equations involving logarithmic and exponential functions correctly to find the coordinates of stationary points, demonstrating good algebraic skills
- provided the value of t when given (x,y) to evaluate the gradient for a function defined parametrically
- provided the equation of a normal at a point on the y-axis
- solved a simple related rates problem
- identified the variable and the constant in a function, and differentiated it accordingly
- demonstrated reliable and accurate algebra skills when solving problems
- demonstrated the procedure to set up models for optimisation questions, in terms of one variable, before correctly differentiating it.

Candidates who were awarded **Achievement with Excellence** commonly:

- demonstrated excellent algebraic skills when solving problems
- provided the gradient of a tangent to a curve defined parametrically given the *x* and *y* coordinates of the point of intersection of the tangent with the curve
- modelled the volume of a cylinder, find its derivative, maximise the volume for the given object, and then prove it was maximum, using either a first or second derivative test
- provided the point of intersection of a tangent with a curve, given the equation of the curve and a point on the tangent that was not on the curve
- provided solutions and justified the rejection of values that were not a solution of the original equation
- used the quotient rule to find the derivative of a rational function, and solve the resulting quadratic inequation to find the values of *x* for which the function was increasing
- identified the variables and the constants in each situation so that they could form a model with one variable, which they could differentiate
- differentiated a function involving multiple brackets with fractional powers, and demonstrated excellent algebraic skills while completing the proof that the edge of the table would have maximum illumination for the given h value
- completed proofs using logical, clear working.

91579: Apply integration methods in solving problems

Examinations

This paper gave candidates multiple opportunities to display their understanding of the material outlined in the standard. Namely:

- integrating power, polynomial, exponential (base e only), trigonometric, and rational functions
- reverse chain rule, trigonometric formulae
- rates of change problems

- areas under or between graphs of functions, by integration
- finding areas using numerical methods, e.g. the rectangle or trapezium rule
- differential equations of the forms $y' = f(x)$ or $y'' = f(x)$ for the above functions or situations where the variables are separable (eg $y' = ky$) in applications such as growth and decay, inflation, Newton's Law of Cooling and similar situations.

While the ability to integrate the types of functions listed in the first bullet point above is critical to success, so is having the algebra skills required to solve the problem. These algebra skills are necessary for all three external standards in the Calculus course and their constant practice and reinforcement should be a focus for all candidates.

Observations

Candidates should be strongly discouraged from only attempting the "Excellence" part of each question. It was very common for candidates who did this to make an error on one or more of these.

Teachers need to stress with candidates the correct use of constants.

For example, $y/2=tanx+c$ was often followed by $y=2tanx+c$, which is incorrect.

The *c* in the second expression does not have the same value as the *c* in the first expression. If the *c* is calculated from the second line it will be incorrect when applied to the original equation. The second line should be $y=2tanx+2c$ or $y=2tanx+d$.

When solving a differential equation of the typed $y/dx=(f(x))/y$, most candidates correctly separate the variable to get $\int ydy=\int f(x)dx$.

However, a large percentage of candidates integrated the left-hand side of the equation to *y* rather than x^2/y.

Integrating $1/\sqrt{V}$ to $\sqrt{V}/2$ rather than $2\sqrt{v}$ was quite common in question 1 (d). This is an incorrect integration, so no credit could be given for this question, even for candidates who managed to arrive at the correct solution.

Grade awarding

Candidates who were awarded **Achievement** commonly:

- integrated basic functions correctly

- manipulated functions into a form which could be integrated
- attempted the majority of questions, in particular integrating any functions in the paper that needed to be integrated
- used trapezium rule
- provided the constant of integration given the necessary information.

Candidates whose work was assessed as **Not Achieved** commonly:

- did not demonstrate understanding required for integrating power, polynomial, exponential, trig, and rational functions
- did not algebraically rearrange or manipulate a function so that it was in a form which they could then integrate
- did not use a numerical method without difficulty
- confused differentiation and integration methods.

Candidates who were awarded **Achievement with Merit** commonly:

- integrated a rational function by using either long division or substitution
- used trigonometric identities correctly
- solved differential equations by separating variables, including calculating the arbitrary constant
- recognised when to use the natural logarithm to integrate
- recognised that ∫ydy is y^2/2 , not *y*
- recognised that integrating a derivative function gives the function, and that to find the area under that function a second integration is required.

Candidates who were awarded **Achievement with Excellence** commonly:

- communicated each step of a 'proof' question with sufficient detail
- carried out appropriate algebraic manipulations
- recognised the relationships between equations and graphs
- demonstrated understanding which letters where variables and which were constants when integrating

- formed a differential equation for a contextual problem, correctly separated the variables, and then used the appropriate information to find the value of the constants.

LIVE

Mathematics and Statistics subject page

Previous years' reports

2020 (PDF, 230KB)

2019 (PDF, 152KB)

2018 (PDF, 125KB)

2017 (PDF, 48KB)

2016 (PDF, 255KB)