2022 NCEA Assessment Report

Subject: MATHEMATICS AND STATISTICS

Level: 1

Standards: 91028, 91031, 91037

Part A: Commentary

The examination provided opportunities for candidates to demonstrate knowledge and understanding of the whole content of the syllabus relevant to AS 91028, AS 91031, and AS 91037. Candidates, however, are expected to have understanding and knowledge of other wider aspects of the curriculum.

Candidates will be advantaged by preparing for the examination with a thorough review and analysis of several years past examinations to familiarise themselves with possible content, expectation and the style of wording of the assessment.

These are all available from the NZQA website:

www.nzqa.govt.nz/ncea/assessment/search.do?query=math&view=files&level=01

Question parts may follow-on from each other and be linked, so a candidate should be actively looking for this connection and, if necessary, referring to earlier pages in the question booklet.

Question parts are not necessarily in order of difficulty, and each question part is likely to contain multiple levels of award. Candidates are encouraged to attempt all parts of all questions.

Candidates must read the questions carefully to ensure that the content of the context is understood fully. Candidates are encouraged to use bullet-point responses so that they can check-off that they have answered all aspects of the question. Often a question will provide guidance to the candidate as to how much detail is expected in a response, e.g. “Give at least three different supporting statements”. Further direction is provided by the examiner with comments such as “Justify your answer”. Candidates may not receive any reward for correct answer-only responses and are unlikely to receive anything better than achieved, at best.

The examination requires the candidate to have access to a scientific calculator (or graphical calculator), which will enhance and support their performance. However, candidates need to remember that answers written directly from a calculator are likely to be awarded an achieved grade for that question part, at best. Candidates should always support their answer with appropriate working and justification.

Each individual Achievement Standard will require a range of required methods and procedures, taken from Level 6 of the NZ curriculum. A combination of these may be required in any part of any question for the award of any level of achievement. Evidence of relational and abstract thinking may be demonstrated by linking of these, leading towards all levels of achievement within that part of the question.

91028: Investigate relationships between tables, equations and graphs

Examination

The examination contained three questions, each with multiple parts that covered the requirements of the 2022 assessment specifications and achievement standards at Level 6 of the New Zealand curriculum. Candidates were required to answer all parts of all questions.

Within forming and drawing the relationship between linear, quadratic, and exponential functions, candidates were required to comment on and interpret these results, using equations, to solve problems within context. Candidates were rewarded appropriately if they had prepared themselves thoroughly with regards to all aspects of tables, equations, and graphs in relation to linear, quadratic, and exponential graphs. Candidates should be familiar and confident with the features of all linear, parabola, and exponential graphs.

Question One focussed mainly on two separate aspects. One part was related to a spiral pattern that resulted in a parabolic, quadratic discrete function that needed to be graphed. The other main component of this question related to the costings of two courier companies. Candidates needed to interpret the information in context, producing two piece-function linear graphs. The focus was then on comparing the costs of the two companies and interpreting which company was preferred, according to the weights of the parcels needing to be delivered. The higher grades were attained by how well the candidate compared, communicated, and commented on the costs of the two courier companies. For the highest grades, candidates needed to demonstrate knowledge and understanding of detailed and accurate drawing, equations, as well as domains in their communication and interpretation of the situation.

Question Two focussed mainly on the analysis and interpretation of a quadratic function to model throwing balls out of an upstairs window in a tall building. The model described the path of various balls as they were thrown from a window until the balls hit the ground. Candidates were required to form various models from the given context and then analyse them using their graph knowledge, understanding, and interpretation. Predictions regarding the distance travelled or the height component of the balls were enabled by relation tables, equations, and parabola graphs.

Question Three focussed on using the analysis and interpretation of two exponential function models. The models were representing the cooling of a cup of coffee and a cup of tea. Candidates needed to relate tables, equations, and graphs to create and interpret the information provided in context. The objective was to decide when the tea became cooler than the coffee. Candidates had to find the equations of the two situations and then draw the graphs in order to compare them. Alternatively, a candidate could compare the two models by analysing values in a table. Realistically, using equations for the comparison is at not at Level 6 of the curriculum, so this approach was not expected or required. Various approaches were possible, but the candidate was rewarded with effective and clear communication applied to the requirements of this Achievement Standard.

There was also the opportunity to display knowledge of finding the equation of a simple linear, parabolic, and exponential graphs.

Observations

Success for a candidate on this achievement standard will require students to be able to work with, linking and commenting upon, all three aspects of tables, equations, and graphs.

Candidates need to be familiar with and confident in applying aspects in all of linear, quadratic and exponential functions.

Candidates need to be able to create tables and graphs and equations from a problem in context. The application of their graphs knowledge and understanding will refer to context in a variety of situations. These interpretations could include needing to distinguish between whether the graph being drawn should be continuous, discrete or piecewise (step).

Candidates need to be required to distinguish whether the context being analysed is linear, quadratic, or exponential, or a combination of these. Forming the equations of each of these will be a crucial aspect of the assessment. Being confident in their drawing and interpretation is important. For the higher grades, candidates need to apply, interpret, and comment upon a combination of tables, graphs, and equations of linear, quadratic, and exponential functions.

Emphasis is given to clear and effective communication and justification of solutions. Candidates must recognise that transferring answers from a calculator is unlikely to gain higher than the award of achievement. Questions indicate to the candidate that evidence, justification, working are all expected and necessary.

Equations for a function must be given as an equation, and not as an expression i.e. $y = f(x)$.

Grade awarding

Candidates who were awarded **Achievement** commonly:

- were able to form the equation of a linear function, a quadratic function and an exponential function from its graph or from context
- drew a parabolic graph from the context of the information provided
- located the maximum of a quadratic function
- interpreted the asymptote, in context, of an exponential function
- recognised one valid transformation of a parabolic or exponential graph
- produced values in a table from the context of the information provided
- attempted a sufficient quantity of the opportunities available within the examination
- were familiar with and reasonably confident with at least two out of the three main Achievement Standard content, i.e. linear, quadratic and exponential functions
- recognised the relationship between tables and graphs, graphs and equations, and tables and equations.

Candidates whose work was assessed as **Not Achieved** commonly:

- were unable to form the equation of a linear function from its graph
- could not form the equation of a simple quadratic function from its graph

- were not able to complete a table of values from a given pattern in context
- had difficulty to demonstrate their knowledge because the context confused them
- did not complete enough of the available opportunities in the examination.

Candidates who were awarded **Achievement with Merit** commonly:

- derived the equation of a quadratic function using the data provided in a table of values
- derived the equation of a quadratic function from a given graph where the coefficient of x^2 is not unity
- derived the equation of a linear function using the information in context
- recognised that a parabolic graph, in a given context, should contain discrete values only, and demonstrate this with an appropriate graph
- formulated a piece-wise linear graph from given information in context
- recognised more than one valid transformation of a parabolic or exponential graph
- formulated an exponential equation from given information in context
- interpreted, link and comment upon pairs of relationships of between tables, equations and graphs.

Candidates who were awarded **Achievement with Excellence** commonly:

- linked and demonstrated deeper understandings of the links and relationships between tables, equations, and graphs in context, and interpret and comment on these by describing a variety of features of graphs
- formed and solved quadratic equations to solve problems, using a breadth of confident algebra skills and / or graphical methods
- demonstrated a clear and deep knowledge and understanding of all aspects of linear, quadratic and exponential graphs, in context
- clearly communicated and interpreted, using equations, graphs, tables, and domains, linear functions in context
- demonstrated clear and precise justification of their solutions
- interpreted graphically the intersection between two exponential functions in context
- combined effectively deeper understanding of all aspects of equations, tables and graphs in order to interpret and solve problems.

91031: Apply geometric reasoning in solving problems

Examination

The examination contained three questions, each with multiple parts. Candidates were required to answer all parts of all questions. The questions covered the requirements of the 2022 assessment specifications and achievement standard at Level 6 of the New Zealand curriculum.

Question One focussed on diagrams related to a variety of differing contexts. Applications of Pythagoras' Theorem and trigonometry were utilised within the various question parts. There were also question parts related to circle geometry and angles in polygons. Some of the question parts required relational thinking and abstract thinking to be able to interpret the situations present, requiring multiple steps of working and interpretation. The single excellence component required the candidate to produce a generalisation of an angle in polygons situation. Candidates were required to be able to produce their solutions in an appropriate correct mathematical manner.

Question Two focussed on solving and analysis of problems, involving the knowledge of and use of the various geometric circle theorems. One of the question parts required the candidate to recognise and utilise similar triangle knowledge and understanding. There was also an opportunity for candidates to solve a problem requiring deeper extended abstract thinking, requiring the use of trigonometry with a generalised constant included. Success in this question part rewarded a candidate with excellence in this question.

Question Three focussed on further opportunities for displaying knowledge and understanding in solving problems involving straight line geometry, circle geometry, Pythagoras' Theorem, and trigonometry. One of the question parts, based on circle geometry, also included the need to find an angle including the use of an unknown constant, requiring to display relational thinking. The final part of this question assessed the candidates' knowledge with respect to the use of bearings linked to Pythagoras' Theorem and trigonometry. Finding both the required distance and bearing related to the context of the positions of three ships led to the excellence award in this question.

Observations

Candidates' success within this Achievement Standard requires knowledge and understanding of all aspects of the components i.e. Pythagoras' Theorem; trigonometry; straight line geometry; circle geometry; similar triangles. All three questions aimed to include aspects of all the content of this achievement standard.

Problems will aim to link these various aspects so that candidates will be required to be confident with all aspects.

All diagrams are always drawn "not to scale" so candidates should not make assumptions based on how the diagram looks, but should respond based upon geometric reasoning and interpretation. The candidate must not assume any information not provided in the question without deciding, with justification, if that information is actually true or not. Students are encouraged to use the diagram for their own advantage by drawing on extra lines not already provided, and by labelling any information evaluated onto the diagram. This is for the benefit, however, of the candidate and not the examiner.

Candidates should be aware that the presentation of their solutions need to be mathematically correct at all levels of attainment. Methods which are not presented in the necessary formal manner will not be awarded with the respective grades.

Candidates are encouraged to use the labelling provided in the question to help them produce their solution in a clear and logical manner. The labelling provided is given as a help to the candidate.

Candidates should develop the habit of self-checking to decide if their solutions are sensible, and appropriate solutions to the problem given.

Candidates will need the formal geometric reasons to provide the justification of their solutions. When the question stipulates that "justify your answer with clear geometric reasoning", then the examiner requires each step of the solution to have a geometric justification. Solutions that incorporate working that includes Pythagoras' Theorem and / or trigonometric methods should similarly communicate their working and justification in a clear and mathematically correct manner.

Evidence can be obtained from every question part which are not necessarily in order of difficulty within a question. Candidates need to be aware that the application of their knowledge and understanding will be in context in a variety of situations.

Grade awarding

Candidates who were assessed as **Achievement** commonly:

- attempted most parts of all questions, but did not provide sufficient evidence of their working and so only gained credit according to "correct answer only"
- possessed knowledge in Pythagoras' Theorem, trigonometry, and straight-line geometry but were not confident with the necessary circle geometry theorems
- recognised a problem involving similar triangles, and were able to apply that knowledge in solving a problem
- did not transfer their working added to the diagram provided into a written response
- determined the values of missing angles using geometric reasoning knowledge, but were not able to support this with the formal geometric reasoning behind their decisions.

Candidates who were awarded **Not Achieved** commonly:

- did not attempt a sufficiently high proportion of the opportunities available throughout the entire paper
- were not able to recognise and select the appropriate method that was required in a specific question part
- could not distinguish between the requirements of differing methods applying Pythagoras' Theorem, finding the length of a hypotenuse or alternatively a shorter side
- could not select the correct trigonometric ratio necessary to solve a problem
- did not recognise that Pythagoras' Theorem and trigonometric methods can only be applied in right-angled triangles
- did not have sufficient knowledge of the geometric circle theorems
- did not support their answer with appropriate and correct justification
- used Pythagoras' Theorem or trigonometry in non-right-angled triangles
- did not recognise that particular triangles in a circle were isosceles triangles as two of the sides were radii of the circle.

Candidates who were awarded **Achievement with Merit** commonly:

- produced a solution to a geometric reasoning problem, requiring multiple steps of thinking, producing a result in a systematic, formal, mathematical manner
- demonstrated in depth knowledge of all aspects of the material included in this Achievement Standard i.e. Pythagoras' Theorem, trigonometry, straight line, and circle geometry, similar triangles, angle sums of polygons

- were confident in the use and application of problems involving similar triangles
- applied geometric reasoning to a problem involving bearings
- supported their solutions with a systematics and justified formal solution
- utilised some algebraic skills in the solution of generalised geometric problems.

Candidates who were awarded **Achievement with Excellence** commonly:

- solved a problem involving multiple steps while communicating their method and reasoning in a precise and clear manner
- applied geometric reasoning to a problem involving distances and bearings, presenting the solution in a precise and clear manner
- incorporated algebraic skills and knowledge into solving a geometric reasoning problem, establishing an algebraic relationship.

91037: Demonstrate understanding of chance and data

Examination

The examination contained three questions, each with multiple parts. Candidates were required to answer all parts of all questions. The questions covered the requirements of the 2022 assessment specifications and achievement standard at Level 6 of the New Zealand curriculum. Within reading and interpreting statistical representations or analysing statistical investigations, candidates were required to comment on the appropriate and / or misleading aspects of graphical displays of data or statistical information provided in other formats.

Question One focussed on the analysis and interpretation of a time-series graph. The information was representing the maximum and minimum temperatures in Auckland. This is a context with which most students would have been familiar. The data provided was authentic, and consequently the data did not actually have a repeating pattern. Candidates need to be aware that real data need not necessarily present itself in such an idealistic matter. Question parts directed candidates in such a way as to ensure that no reference to a repeating pattern was expected. The context was explained to candidates to clarify the context in the introduction to the question and graph.

Students needed to know how to read and interpret information from a time series graph, discuss long-term trends and unusual features, and to compare the 2009 data with the 2019 data represented in the time series graphs.

The second part of the question focussed on probabilities and claims based on a two-way table containing average temperatures in Auckland in 2019. Candidates are encouraged to recognise the difficulties and necessary precautions, with regard to making accurate and appropriate predictions from given data.

Question Two focussed on the analysis and interpretation of two types of graphs. In the first part of the question, candidates were provided with a statistical graph which represented temperatures which were above or below a baseline average temperature. The introductory paragraph explained the usefulness and interpretation of this style of graph, which may have been unfamiliar to many students. Students are encouraged to read this information carefully, which aims to clarify and inform the candidates. Subsequent parts required the candidates to interpret and identify statistical features from this graph.

In the second part of the question, candidates were provided with a bivariate data graph showing the relationship between the temperatures in Auckland and Wellington from 1966 to 2019. Candidates needed to show their knowledge and understanding of what this type of graph represents. The aspects of the bivariate data being investigated were interpreting different features, lines of best fit, and commenting on how useful the graph was in making predictions. This question also contained aspects related to conditional probability. Candidates are reminded that evidence of their final answer should be provided as well as sufficient accuracy if the response is being given as a decimal.

Question Three focussed mainly on the analysis and interpretation of multivariate data provided in a box and whisker graph. Students were assessed on their knowledge and understanding of inter-quartile range, comparing distributions, making an inference from the box and whisker graph.

There was a further opportunity for students to display their knowledge and understanding of probability with information provided in a graphical representation. The higher rewards were possible with the candidate displaying their capability of using conditional probabilities.

Observations

Candidates need to be aware that essay-style responses are not required. Bullet-point answers are acceptable and may help the candidate ensure that they have answered all aspects of the question.

When interpreting statistical graphs, knowledge and understanding of what the graphs is communicating is essential. For this achievement standard examination, time series, bivariate data graphs, and multivariate data graphs should be expected. Other statistical graphs, however, may appear giving the candidate the opportunity to explore and interpret these.

Candidates should be familiar with the terminology of “statistical models”, as questions will use this phraseology in introducing the context of a set of data.

Candidates may be expected to find conditional probabilities using an informal approach. The use of a probability tree diagram is not expected or required but could be utilised by a candidate.

Candidates need to be aware, when using decimals, that using sufficient accuracy in rounding the answer is important especially when the candidate has not shown evidence from where the decimal appears. Candidates are encouraged to use fractional answers and, as always, show full and clear justification to support the final answer.

There are comments and directions in the questions that should help guide the candidates towards the expectation of a full answer. Candidates need to utilise these helpful directions in order to provide the depth of quality and required content of the response that is expected for a higher-grade reward.

E.g. “describe and interpret at least two features”, “justify your answer using statistical reasons”, “give at least two different statements to support your decision”, “describe at least three different key features”, “justify your prediction”, and “provide evidence from the graph”.

A significant number of candidates did not support their decision with appropriate justification, which the question required and stated. Often a response with no reasoning or justification will not be rewarded by the examiner.

Grade awarding

Candidates who were assessed as **Achievement** commonly:

- calculated simple probabilities from interpreting data provided in various formats
- calculated simple probabilities from a two-way table
- realised that responses required justification
- provided valid statistical responses when assessing a claim made in the data provided
- calculated and interpreted the inter-quartile range (IQR) from a box and whisker graph
- interpreted multivariate data from a contextual situation
- provided one feature when comparing two distributions provided in a box and whisker graph
- interpreted a time series graph and a bivariate data graph.

Candidates who were awarded **Not Achieved** commonly:

- did not attempt a sufficient quantity of the opportunities made available in the examination
- did not identify similarities or differences comparing two distributions represented in a box and whisker graph
- did not describe or interpret features in a bivariate data graph
- did not have basic probability knowledge
- did not interpret a time series graph
- produced answers with no justification.

Candidates who were awarded **Achievement with Merit** commonly:

- identified at least two features in a bivariate data graph
- commented on the appropriateness of a bivariate data model
- provided justified responses to the usefulness of a bivariate data model
- described similarities and differences between the features of two multivariate distributions presented in a box and whisker graph
- made an appropriate inference from two multivariate distributions presented in a box and whisker graph
- interpreted and identified two statistical features in an unfamiliar setting
- made justified comments on claims based on data provided in a two-way table
- compared and commented upon the comparisons between two time series graphs
- calculated relevant probabilities in a conditional probability problem.

Candidates who were awarded **Achievement with Excellence** commonly:

- demonstrated abstract thinking in interpreting and reflecting upon a probability-related situation
- were confident in all aspects of a multivariate data box and whisker graph, knowing about multiple factors of centre, shift, shape, spread whilst connecting this to making an appropriate inference
- provided at least three different features when comparing two distributions provided in a box and whisker graph
- interpreted and identified at least three statistical features in an unfamiliar setting
- provided justification for their comments, including numerical data and calculations
- demonstrated insight by analysing a time series graphs in depth and with a high level of understanding and interpretation, backed up with appropriate numerical calculation and justification
- calculated more advanced conditional probability calculations.