2024 NCEA Assessment Report

Subject:	Calculus
Level:	3
Achievement standard(s):	91577, 91578, 91579

General commentary

The 2024 papers were in a similar format to those examinations from previous years. Candidates are encouraged to ensure that they make previous examinations papers a part of their end-of-year exam preparation, studying and analysing the content, style, and format. Candidate performance was of a similar standard to previous years. Candidates displayed the knowledge and understanding to solve problems using conventional methods, but also using innovative methods. It is disappointing to see, however, that there were also candidates who made fundamental and basic mathematical errors of various natures, which showed a lack of the deeper, necessary, understanding of some aspects of the mathematics at this level. Careless methods and errors often resulted in candidates missing out on success.

Candidates are encouraged to set out their working in a clear, logical, and systematic manner. This is a real advantage when solving the problems, providing the required evidence for the marker as well as helping the candidate develop their strategy as they progress through the problem. This is beneficial, particularly in problems that require an extended chain of reasoning. This will also enable the candidates to "self-check" as they progress through their solution in order to avoid the careless errors.

Candidates must ensure that they carefully read the information provided in the question, which often provides helpful guidance and critical information, as well as helping the candidate recognise the requirement of the question. Unfortunately, many candidates were not able to be rewarded as they did not fully answer the demands and expectations of the question (e.g., solving an equation for x, in terms of k; but actually, in error, solving the equation for k, in terms of x).

Candidates, whatever their target grade, are encouraged to attempt all parts of all questions. All question parts provide opportunities for success. It is not advisable for those candidates attempting to gain an overall Excellence grade to only attempt the questions that they believe to be Excellence questions. This strategy is not advisable as an introduced error or misinterpretation often leads to little reward. A large proportion of candidates who adopted this approach ended up with results well below their target grade. The examiner carefully ensures that there is sufficient time available in the examination to attempt all question parts. Candidates who have pre-planned a strategy for their success can be caught out when the more challenging questions are not familiar to them. Conversely, the weaker candidates often can gain valuable points in a question by completing part answers for the more challenging questions.

Candidates who were not successful often attempted only one or two parts of each question. There are many opportunities to gain credit for correct working in all problems in all question parts, even though some candidates may struggle to solve the whole problem completely. Candidates are encouraged to answer at least a portion of all question parts of the majority of the paper in order to maximise their chance of success. A candidate should start their solution, perhaps not recognising the complete pathway through the question, and then often will find that the most appropriate strategy will reveal itself later in the solution.

Candidates need to understand that “Correct Answer Only” responses, directly from the graphical calculator, do not provide evidence of relational thinking. The marker can only reward a candidate when there is evidence at the required level of the mathematics being scrutinised. Omitting steps, with the use of a graphics calculator, is not an advised strategy.

Candidates are reminded that the square root of a number has two solutions, the positive and negative value. The negative value should not be forgotten or ignored, but included or rejected, as appropriate.

Candidates should ensure that any incorrect working or non-relevant part-solution should be crossed out, so that the marker is aware which solution is the one to be considered, and hence the marker will not include this in their analysis of the response.

Candidates should ensure that a sufficient degree of accuracy is used throughout a solution when numerical values are being introduced. The rounding of the final answer should appear only at the final stage of the solution.

Candidates need to be confident in using their calculator efficiently, effectively, and correctly, to support their working (e.g., evaluating and substituting radian values into trigonometric functions that involve exponents). A problem is not fully solved until the final correct answer has been attained.

Report on individual achievement standard(s)

Achievement standard 91577: Apply the algebra of complex numbers in solving problems

Assessment

This examination provided candidates with multiple opportunities to display their understanding of the material outlined in the standard, namely:

- quadratic and cubic equations with complex roots
- Argand diagrams
- polar and rectangular forms
- manipulation of surds
- manipulation of complex numbers
- loci
- de Moivre’s theorem
- equations of the form $zn = r \text{ cis } \theta$, or $zn = a + b\text{i}$ where a, b are real and n is a positive integer.

A study of past examination papers and schedules will show that there are many skills that are regularly assessed. Problems related to the many and varied aspects of complex algebra need to be thoroughly understood and learned for the higher levels of success in this achievement standard. In particular, candidates should be thoroughly prepared for various questions that require the knowledge and application of de Moivre’s theorem.

The requirement to show an answer in terms of a specific variable or in a specific form is not understood or is ignored by some candidates. If the question stipulates that the solution needs to be given in a particular format, then this guidance should be followed for full credit (e.g., giving a complex number in the rectangular form or polar form).

Many candidates lacked the necessary knowledge and understanding to form the expression to solve the problem that resulted, or understand the algebra required to make progress towards an answer.

Candidates should recognise that an explanation, comment, or interpretation should be included, as well as relevant calculations, as they are a necessary and important part of the solution.

Commentary

There are fundamental skills that candidates who target success in this standard must have the confidence in recognising and using.

The ability to multiply and divide complex numbers in both rectangular and polar form is the most fundamental aspect of this standard. The need for multiplying by a conjugate fraction was required several times in this examination. It should be a skill that is thoroughly rehearsed by candidates in its various forms.

The skill of understanding and finding the modulus and argument of a complex expression was also required several times in the paper.

Candidates should also be confident about using these skills without resorting to a graphical calculator as there may be constraints set in the question that make the use of a graphical calculator redundant.

Candidates should be very familiar with the important definitions and related terminology that need to be recognised and utilised (e.g., magnitude, modulus, argument, conjugate, Argand diagram).

Application of de Moivre's theorem is another important technique. Candidates need to be very careful with their answers for questions requiring the use of de Moivre's theorem when finding solutions to an equation. Candidates need to ensure they calculate the correct argument when converting the original equation to polar form. If this initial angle is incorrect then consequently so is all the following work. The use of a diagram is encouraged to help identify the correct argument value that will then lead into setting up a general solution with this correct initial angle.

Candidates had trouble representing the modulus correctly, so all consequent evidence in the question was wasted. Using a sketch Argand diagram is recommended to illustrate in which quadrant the point is lying, without relying on a graphical calculator solution.

Grade awarding

Candidates who were awarded **Achievement** commonly:

- evaluated the quotient of two complex numbers in Polar form
- showed an understanding of the argument and modulus of a complex number
- evaluated the modulus of a complex number, with an unknown constant
- evaluated a complex number and plotted it in the Argand diagram
- appied de Moivre's Theorem to simplify a complex number in polar form involving exponents
- recognised how to simplify a complex number by multiplying by the complex conjugate, when the complex number involved an arbitrary constant k
- manipulated a quadratic equation to identify equal real roots, using the discriminant, and solving the resulting quadratic equation
- expanded and simplified an expression involving perfect-square brackets, surds, that needs rationalising, when the surd involved an arbitrary constant p
- identified real and imaginary terms in an expression and could group them correctly
- utilised the factor and remainder theorems to find an unknown constant
- were familiar with and could utilise the formula $y = mx + c$
- attempted a variety of questions
- manipulated complex numbers in either polar or rectangular form
- partly found the cartesian equation of a locus.

Candidates who were awarded **Achievement with Merit** commonly:

- solved an equation involving surds and an unknown constant
- demonstrated understanding of, and could interpret, the modulus and argument, and manipulated an equation involving complex numbers to solve it
- showed that the modulus of a complex number must have a positive value
- accurately found the modulus and argument of a complex number that does not lie in the first quadrant
- solved equations involving surds or imaginary numbers
- used de Moivre's theorem, and applied it correctly to find all of the solutions to a given problem
- demonstrated understanding of the meaning of "purely real" or "purely imaginary" complex numbers and could form the equations that resulted
- substituted $z = x + iy$ into equations, then simplified the resulting equation, separating terms into the real and imaginary parts
- recognised and applied the definitions and terminology of complex numbers
- used appropriate algebraic methods to find complex roots of a cubic polynomial, when given one imaginary root
- formed the cartesian equation of a locus described by a complex equation.

Candidates who were awarded **Achievement with Excellence** commonly:

- used algebra skills to accurately solve an equation, involving quotients of complex numbers, without unnecessary or confusing statements in their working
- recognised when the discriminant should be utilised, and when the quadratic formula should be utilised
- demonstrated problem-solving skills required to group real and imaginary terms, and applied the correct algebra to them
- demonstrated understanding of what the modulus symbol required, and how it could be applied in a formal proof
- explored problems where not all solutions would be valid and solved accordingly
- completed the required proof by making connections between real and imaginary parts, and completing the square
- communicated their thinking clearly and accurately about what they were doing, while completing multi-step problems
- provided clear, logical, and easy-to-follow working, reducing the chance of numerical or algebraic errors
- showed careful thought in recognising when a chosen strategy was not likely to be successful, and then reassessed the chosen method
- displayed abstract thinking in solving problems that were not familiar, making links to the information provided in the question.

Candidates who were awarded **Not Achieved** commonly:

- did not demonstrate basic algebra skills needed to simplify, solve, expand, and factorise
- displayed little understanding of the process for completing the square, theories relating to the discriminant, or quadratic formulae
- did not rationalise a denominator correctly
- were not familiar with the definitions of modulus and argument, and hence could not find the modulus or argument of a complex number
- did not manipulate complex numbers in either Polar or Rectangular form

- converted a complex number from one form to another (rectangular to polar or vice versa) before performing calculations with complex numbers
- did not demonstrate how to convert to polar form without the use of a calculator
- generally relied too heavily on the use of a graphics calculator
- showed no understanding of interpreting or using de Moivre’s Theorems
- showed no understanding of, or how to interpret complex numbers represented on an Argand diagram
- showed no understanding of or how to correctly apply the Remainder Theorem
- made too many basic errors in simplifying and applying algebraic methods
- did not multiply surds together correctly, nor solve equations containing surds
- did not maintain consistency when solving inequations or equations
- did not attempt a sufficiently large proportion of the assessment.

Achievement standard 91578: Apply differentiation methods in solving problems

Assessment

This examination provided candidates with multiple opportunities to display their knowledge and understanding of the material outlined in the standard, namely:

- derivatives of power, exponential, and logarithmic (base e only) functions
- derivatives of trigonometric (including the reciprocal trigonometric) functions
- optimisation
- equations of tangents and normals
- maxima, minima, and points of inflection
- related rates of change
- derivatives of parametric functions
- chain, product, and quotient rules
- properties of graphs (limits, differentiability, continuity, concavity).

Candidates must ensure that they are fully confident in their own ability to be able to differentiate all various forms of the functions, including when the use of the chain, product, and quotient rules are necessary. Many candidates displayed the necessary understanding to be able to interpret the application using differentiation methods, but were let down by errors in their methods of differentiation, including not recognising when product and quotient rules need to be implemented. Success can only be awarded if the applications of the differentiation rules are displayed accurately, error-free, and mathematically correct.

To solve the problems involving application of differentiation, candidates required sound algebra skills, as well as strong knowledge of the various differentiation methods, which come from regular and extensive practice from a wide variety of possibilities. Candidates needed to be confident enough to solve quadratic equations, equations involving the use of exponentials and logarithms, and simple trigonometric equations. Candidates should not rely solely on their graphical calculators to solve such equations, as the inclusion of unknown constants will generally eliminate this pathway.

For all levels of success, a combination of both skills of differentiation and algebra are necessary and essential.

Modelling for optimisation problems continues to be a skill that only the best candidates can handle. Candidates need to work with their supporters to find ways to gain greater confidence in these types of problems. Most candidates were not able to differentiate successfully the function provided in Question Two (e), which involved a mixture of both product and quotient rules. This, therefore, blocked these candidates from being able to successfully apply their resulting equation. Candidates needed a lot more practice in understanding how to deal with creating appropriate models within an optimisation problem.

Many candidates were confused when the problem connected their differentiation and coordinate geometry knowledge (Question Three (e)). It was evident from the errors seen that candidates were not sufficiently confident and experienced in this type of problem-solving, while relying too heavily on the use of the graphical calculator, which made the proof involving exponential-e beyond only the strongest candidates.

Commentary

Candidates are reminded that evidence of clear working and methods must be shown at all levels of the examination. Answers provided without appropriate supporting working are unlikely to be fully rewarded. Candidates should follow the guidance in those questions that stipulate “you do not need to simplify your answer”.

Strong algebraic skills and confidence in all differentiation methods are crucial for success in this standard: expanding, factorising, simplifying expressions, manipulating algebraic fractions, and solving various equations, especially quadratic equations but also equations involving exponential, logarithmic, rational expressions, and trigonometric functions. Similarly, candidates should be aware that the square root of a value will always lead to two possible solutions, one of which may not be valid in a particular question part.

The necessary and effective correct use of brackets and accurate mathematical statements remains a challenge for many candidates. The ability to avoid making careless errors or having an effective self-checking system to eliminate any careless mistakes is important.

Many candidates were unable to differentiate correctly to find the first and second derivatives of the function in Question Two (c), as errors were confused by the necessary use of product rule and function of a function rule.

Candidates frequently omitted a factor of the differentiation, which led to an incorrect differentiation, so no credit could be given for this question part, even for candidates who subsequently managed to continue with their solution.

Candidates needed to communicate formally when solving an equation involving a denominator and equal to 0. Similarly, equations involving exponentials must have working and evidence clearly demonstrated. The marker needs to be shown every step of the process, whereas it is clear candidates rely too much on their graphical calculator for their solutions.

Candidates needed to be aware and knowledgeable regarding completing a proof in an appropriately formal mathematical manner. Many were able to differentiate successfully but were then not able to complete the proof in a sufficiently formal manner (Question Two (c), Question Three (e)).

Grade awarding

Candidates who were awarded **Achievement** commonly:

- used the chain rule, product rule, and quotient rule correctly in combination with power functions, trigonometric functions, parametric functions, and exponential and logarithmic functions
- successfully wrote surd expressions into fractional index form, before differentiating
- solved quadratic, exponential, and power equations resulting from differentiating a function
- provided the conditions, using the derivative, for when a function is decreasing

- used differentiation to find the gradient of tangents and of functions
- solved a kinematics problem, involving differentiation
- used an appropriate model to solve an optimisation problem and then correctly differentiated their model
- found the two components of a related rates of change problem and then correctly differentiated them
- calculated the *x*-coordinate correctly for any stationary points and inflection points
- recognised features of gradient, differentiability, and limit from a piecewise graph
- discarded $x = 0$ as a solution, where appropriate
- showed sufficient algebraic skills to simplify their answer having differentiated a function when required to do so
- confidently and knowledgeably used their calculators to evaluate functions and differentiated functions, including the correct use for trigonometric and exponential functions.

Candidates who were awarded **Achievement with Merit** commonly:

- solved equations involving natural logarithmic and exponential functions correctly to find the coordinates of stationary points and inflection points, demonstrating confident algebraic skills
- demonstrated reliable and accurate algebraic skills and knowledge, with clear and full communication, when solving problems
- were able to use confidently and correctly the product and quotient rules, and chain rule, when differentiating functions
- communicated their solutions distinctly, with clear working, in order to demonstrate their understanding and methods, ensuring that the requirements of the question were fulfilled
- solved a related rates problem, involving the volume of a cone
- used relational thinking to connect coordinate geometry and differentiation knowledge, and apply to gradients of points, tangents, and normals
- demonstrated the procedure to set up models for optimisation questions, in terms of one variable, before correctly differentiating it and then solving the resulting equations
- found second differentials of exponential and trigonometric functions, and continued to use these to prove that a given function was a solution to a given differential equation
- identified a range of values of *x* for when a function is decreasing, necessitating solving quadratic inequalities
- located and identified the nature of stationary points and inflection points, utilising a variety of options of methods.

Candidates who were awarded **Achievement with Excellence** commonly:

- demonstrated high quality algebraic skills when solving problems resulting from differentiating, especially when simplifying expressions
- set up appropriate models and interpreted them correctly to solve the problem using differentiation
- manipulated surds and aspects of exponential functions correctly, resulting from differentiating and evaluating
- confidently found and used the second derivative of functions when solving problems
- demonstrated abstract thinking while applying differentiation methods in solving unfamiliar problems, utilising precise and accurate algebraic methods
- linked differentiation and coordinate geometry methods to find the equation of a tangent at a point of inflection

- used the product and quotient rule to find the derivative of a rational function that included a pronumeral, then successfully connected using the discriminant to find the value of the pronumeral, thus solved the resulting quadratic equation to find the value of x for which the function had a turning point
- formed an area function for a triangle and found the optimal solution to prove the exact maximum area
- completed a proof providing logical, clear working and communicating each step of a 'proof' question with sufficient detail, demonstrating strong and precise algebraic justification
- formed, interpreted and solved problems related to maximums, minimums, inflections, turning-points, and stationary points.

Candidates who were awarded **Not Achieved** commonly:

- were not confident or accurate in recognising and applying the chain rule, product rule, or quotient rule in combination with power functions, trigonometric functions, parametric functions, or exponential and logarithmic functions
- appeared confused about how, when, and which differentiation method should be applied
- did not recognise when it was necessary to apply the product and quotient rules for differentiation
- made careless errors in a solution, reflecting a lack of effective self-checking processes
- did not solve a resulting equation algebraically, even though they may have found the correct derivative
- did not correctly factorise and solve quadratic equations, even though they may have found the correct derivative
- demonstrated poor algebraic skills, such as inaccurate expanding, factorising, cancelling of factors, manipulating algebraic fractions, knowledge of surds, careless errors, and providing only one solution for the square root of a value
- did not recognise features of gradient, differentiability, and limits from a piece-wise graph
- did not directly address the requirements of the question
- made errors in the use of their calculator, especially when evaluating terms involving trigonometric functions, exponential functions
- relied too much on the graphical calculator to provide solutions and evaluations
- omitted the necessary brackets in their solution.

Achievement standard 91579: Apply integration methods in solving problems

Assessment

This examination provided candidates with multiple opportunities to display their understanding of the material outlined in the standard, namely:

- integrating power, polynomial, exponential (base e only), trigonometric, and rational functions
- reverse chain rule, trigonometric formulae
- rates of change problems
- areas under or between graphs of functions, by integration
- finding areas using numerical methods, e.g. Simpson's Rule or Trapezium Rule
- differential equations of the forms $y' = \mathrm{f}(x)$ or $y'' = \mathrm{f}(x)$ for the above functions or situations where the variables are separable (e.g. $y' = ky$) in applications such as growth and decay, inflation, Newton's Law of Cooling, and similar situations.

While the ability to integrate the types of functions listed in the bullet points above is critical to success, so is having the algebra skills required to solve the application problems. These algebra skills are necessary for all three external standards in the assessment and their constant practice and reinforcement, including the essential use of brackets to communicate methods and maintain accuracy of solutions, should be an essential focus for all candidates. The omission of brackets in working often leads to errors in subsequent working, as well as being penalised as being mathematically incorrect.

Candidates must ensure that they are fully confident in their own ability to integrate all various forms of the functions, illustrating their knowledge and strengths in recognising which method is applicable. Many of the candidates displayed the necessary understanding to be able to interpret the application using appropriate integration methods but were consequently thwarted by errors in their methods of integration. Success in this standard can only be awarded if the methods are applied accurately and free of any errors.

Commentary

Candidates are strongly discouraged from only attempting the "Excellence" part of each question. It was very common for candidates who did this to make an error on one or more of these and ultimately under-perform on their intended and desired target. Many candidates enhanced their overall final grade by attempting all parts of all questions, thereby gaining grade u or grade r rewards on the Merit or Excellence question parts.

Candidates are encouraged to use the formal and correct terminology throughout the solution to a problem, using a step-by-step strategy, to avoid careless errors and to guide the candidate through each stage of the necessary working and calculations, including the accurate use of the constant of integration. Within the teaching and learning programme, it needs to be stressed that candidates must correctly use the constant of integration. The constant of integration should not be omitted and its value need not necessarily be zero. Within the formal working while solving a differential equation, care must be taken to use the constant of integration consistently throughout the solution.

Candidates are reminded that evidence of the correct use of integration methods must be clearly demonstrated. Answers from a graphical calculator are unlikely to attain higher than an Achievement grade, at best.

Candidates frequently omitted a factor of the integration which led to an incorrect integration, so no credit could be given for this question part, even for candidates who managed to arrive at the correct solution.

Grade awarding

Candidates who were awarded **Achievement** commonly:

- integrated basic functions correctly, including polynomial functions, rational functions, exponential functions, and trigonometric functions
- successfully wrote surd expressions into fractional index form before integrating
- manipulated functions and expressions into a form which could be integrated
- attempted the majority of questions, in particular integrating any functions in the paper that needed to be integrated
- found an unknown limit, using integration methods
- evaluated simple areas, using integration methods
- solved simple differential equations, finding the particular solution that satisfied the given conditions
- solved simple kinematics problems needing the use of integration methods
- provided and evaluated the constant of integration, given the necessary information.

Candidates who were awarded **Achievement with Merit** commonly:

- integrated a rational function that required necessary manipulation by using either long division or recognition
- used trigonometric identities correctly to apply an appropriate integration method
- solved differential equations by successfully separating variables, subsequently including calculating the arbitrary constant
- recognised when to use the natural logarithm to integrate
- evaluated an area that lay between two functions
- applied integration methods in relation to kinematics
- found an unknown limit using integration methods and solved an equation involving logarithms (base e)
- confidently recognised which integration method could be applied to a specific problem
- were well-rehearsed with regards to certain popular integrations requiring relational thinking
- calculated the required value of the unknown constant, k, so that the two areas under a graph would have equal size
- used formal and accurate algebraic procedures in their solutions, particularly the correct useage of brackets in appropriate places.

Candidates who were awarded **Achievement with Excellence** commonly:

- carried out appropriate algebraic manipulations to attain an expression that could then be recognised to fit into a suitable standard integration formula
- recognised the need to use appropriate trigonometric formulae so that an area could be evaluated using integration methods
- demonstrated understanding to use correct and appropriate mathematical statements within a systematic chain of logical communication of their method
- managed negatives when manipulating and integrating expressions
- demonstrated the ability to form and solve a differential equation for a contextual problem, related to Newton's Law of Cooling.

Candidates who were awarded **Not Achieved** commonly:

- lacked sufficient knowledge and understanding to recognise the appropriate method and consequent accurate integration when integrating power, polynomial, exponential, trigonometric, and rational functions
- believed that all rational expressions led to an integration involving $\ln(f(x))$, e.g., incorrectly omitting the factor of 4 in the integration of $5 / 4x - 3$
- did not algebraically rearrange, simplify or manipulate a function so that it was in a suitable form which could then be integrated
- did not recognise how integration could be used to evaluate an area
- were not sufficiently confident and knowledgeable with the rules of indices so that errors appeared when attempting to integrate functions needing rearranging first, e.g., rewriting $1/12y^2$ incorrectly
- did not correctly rearrange $\sqrt{x}$ into an alternative appropriate format involving exponents that could then be integrated
- confused differentiation and integration methods
- did not separate the variables in a differential equation
- omitted a factor of the integration which consequently led to an incorrect integration
- relied too heavily on the use of their graphical calculator when evaluating definite integrals and omitted the necessary evidence of the integration method used.