2025 NCEA Assessment Report

Mana Tohu Mātauranga o Aotearoa
New Zealand Qualifications Authority

Subject:	Calculus
Level:	3
Achievement standard(s):	91577, 91578, 91579

General commentary

The 2025 papers once again were in a similar format to those examinations from previous years. Thus, candidates who availed themselves by scrutiny of the previous years' examinations were well prepared for success in this examination as similar skills to those above are regularly assessed.

Candidates are encouraged to ensure that they make previous examination papers prominent in their preparation, studying and analysing the content, style, and format. The performance of the candidates was of a similar standard to that in previous years. Candidates displayed the knowledge and understanding to be able to solve problems using conventional methods, but markers are always interested to see some candidates using innovative methods. It is also always disappointing to see, however, that there were also candidates who made fundamental and basic mathematical errors, of various natures, which showed a lack of deeper necessary understanding of some aspects of the mathematics at this level. It is frustrating for both the examiner and candidates to miss out on success whilst providing evidence of methods and processes that include algebraic, numeric, or calculator errors, which preclude candidates reaching the desired correct solution.

Common pieces of advice made relevant to all three standards are:

- Candidates are encouraged to set out their working in a clear, logical, and systematic manner. This is a real advantage when solving the problems, providing the required evidence for the marker, as well as helping the candidate develop their strategy as they progress through the problem. This is beneficial particularly in problems with which require an extended chain of reasoning. This will also enable the candidates to "self-check" as they progress through their solution in order to avoid the careless errors.

  Candidates should ensure that they carefully read the information provided in the question which often provides helpful guidance and critical information, as well as helping the candidate recognise what is the requirement of the question. Unfortunately, many candidates are not able to receive the full reward as they do not fully answer the demands and expectations of the question.

  E.g. the question asks to solve an equation for x, in terms of k; but actually, in error, the candidate has solved the equation for k, in terms of x.
- Candidates, whatever their target grade, are encouraged to attempt all parts of all questions. All question parts provide opportunities for success at all levels. It is similarly not advisable for those candidates with potential to gain an overall Excellence grade to attempt only the questions that they believe to be 'excellence' questions. This strategy is not advisable, as an introduced error or misinterpretation of the question often leads to little reward. It continues to be a major disappointment and of concern that a large proportion of candidates who adopted this approach attained results that were well below their target grade. The examiner and their team carefully ensure that there is sufficient time available in the examination to attempt all question parts. Candidates who have a pre-planned strategy for their success can be caught out when the more challenging questions are not familiar or accessible to them. Conversely, the weaker candidates

often can gain valuable points in a question by completing part answers for the more challenging questions.

- Candidates who were not successful often attempted only one or two parts of each question. There are many opportunities to gain credit for correct working in all parts of all question parts, even though some candidates may struggle to solve completely the whole problem. Candidates are encouraged to answer at least a small portion of all question parts of the majority of paper in order to maximise their chance of success. A candidate should start their solution, perhaps not fully aware of the complete pathway through to completing a solution to the question, and then often will find that the most appropriate strategy will reveal itself later in the solution.
- Candidates need to understand that "Correct Answer Only" responses, directly from the graphic calculator, do not provide evidence of relational thinking. The examiner can reward a candidate only when there is evidence at the required level of the mathematics being scutinised. Omitting steps, with the use of a graphics calculator, is not an advised strategy. Candidates will notice from their analysis of past assessments, that many questions are designed in such a manner to preclude a candidate relying on their graphical calculator to solely solve a problem. A candidate, however, should be confident in their use of their graphical calculator, as there will be many benefits in supporting their solution which demonstrating the necessary evidence.
- Candidates are reminded that the square root of a number has two solutions, the positive and negative value. The negative value should not be forgotten or ignored, but included or rejected, as appropriate. In particular, in AS 91577, Question Three (c), many students did not check the validity of both of the answers calculated, to see that actually there was only one true solution.
- Candidates should ensure that any incorrect working or non-relevant part-solution should be crossed out so that the marker is aware which solution is the one to be considered, and hence the marker will not include this in their analysis of the response. When the candidate submits more than one solution, then the examiner will not select the better solution on behalf of the candidate.
- Candidates should ensure that a sufficient degree of accuracy is used throughout all steps of the solution when numerical values are being introduced. The rounding of the final answer should only appear at the final stage of the solution.
- Candidates need to be confident in using their calculator efficiently, effectively, and correctly, to support their working.

  E.g. Many candidates incur calculator errors whilst evaluating and substituting radian values into trigonometric functions, especially when this involves exponents. A problem is not fully-solved until the final correct answer has been attained, and the grade is not attained until that stage.

Report on individual achievement standard(s)

Achievement standard 91577: Apply the algebra of complex numbers in solving problems

Assessment

This examination provided candidates with multiple opportunities to display their understanding of:

- quadratic and cubic equations with complex roots
- Argand diagrams
- polar and rectangular forms
- manipulation of surds
- manipulation of complex numbers
- loci
- de Moivre's theorem

- equations of the form $z^n = r \text{ cis}\theta$, or $z^n = a + b\text{i}$ where a, b are real and n is a positive integer.

Commentary

The requirement to show an answer in terms of a specific variable or in in a specific form is not understood or is ignored by some candidates. If the question stipulates that the solution needs to be given in a particular format, then this guidance should be followed for full credit.

E.g. giving a complex number in the rectangular form or polar form.

Many candidates lacked the necessary knowledge and understanding to form the expression to solve the problem that resulted, or understood the algebra required to make progress toward an answer.

Candidates should recognise that an explanation, comment or interpretation should be included, as well as relevant calculations, as they are a necessary and important part of the solution.

There are fundamental skills that candidates who target success in this standard must have the confidence in recognising and using.

The ability to multiply and divide complex numbers in both rectangular and polar form is the most fundamental aspect of this standard. The need for multiplying by a conjugate fraction was required several times in this examination. It should be a skill that is thoroughly rehearsed by candidates in its various forms.

In Question One (b), candidates failed to solve a quadratic equation with unknown coefficients, either by using the quadratic formula or by completing the square. This seems to imply that too many candidates are relying too heavily on the use of their graphical calculator for their solving.

In Question One (c), many candidates misunderstood what a proof requires, attempting instead to find values of k that give real roots

In Question One (d), candidates frequently missed that the argument was negative and gave the positive value instead.

Question One (e), contained two surds and a constant term. Many candidates attempted to remove the surds by squaring each term separately, which is a fundamental algebraic skill.

The skill of understanding and finding the modulus and argument of a complex expression was also required several times in the paper.

Candidates should also be confident about using these skills without resorting to a graphical calculator, as there may be constraints set in the question that make the use of a graphical calculator redundant.

Candidates need to be very familiar with the important definitions and related terminology that need to be recognised and utilised accurately

E.g. magnitude, modulus, argument, conjugate, Argand diagram.

Application of de Moivre’s theorem is another important technique. Candidates need to be very careful with their answers for questions requiring the use of de Moivre’s theorem when finding solutions to an equation. Candidates need to ensure they calculate the correct argument when converting the original equation to polar form. If this initial angle is incorrect then consequently so is all the following work. The use of a diagram is encouraged to help identify the correct argument value which will then lead into setting up a general solution with this correct initial angle.

Candidates also had trouble representing the modulus correctly, so all consequent evidence in the question was wasted. Using a sketch Argand diagram is recommended to illustrate in which quadrant the point is lying, and without relying on a graphical calculator solution.

Grade awarding

Candidates who were awarded **Achievement** commonly:

- utilised the factor and remainder theorems to find unknown constants
- manipulated complex numbers in both polar and rectangular forms, including converting between these forms
- solved quadratic equations using the quadratic formula or by completing the square, and could simplify surds and rationalise denominators
- manipulated quadratic equations to enable the use of the discriminant, and understood that real roots require the discriminant to be greater than zero
- demonstrated understanding of the argument and modulus of complex numbers, including evaluating and plotting them on the Argand diagram – even when not in the first quadrant
- applied De Moivre's Theorem to simplify or raise complex numbers to a power in polar form
- correctly identified and grouped real and imaginary terms in expressions
- recognised how to simplify complex numbers by multiplying by the complex conjugate
- expanded and simplified quadratics, including those with surds and / or imaginary values.

Candidates who were awarded **Achievement with Merit** commonly:

- solved equations involving surds or imaginary numbers, and checked the validity of solutions
- showed that the modulus of a complex number must have a positive value, and demonstrated understanding of the modulus in problem solving
- used de Moivre's theorem correctly to find all solutions to a given problem
- demonstrated understanding of "purely real" or "purely imaginary" complex numbers, and could form the resulting equations
- used appropriate algebraic methods to find complex roots of a cubic polynomial, when given one imaginary root, and found the remaining solutions and unknown coefficients
- explored problems where not all solutions would be valid and solved accordingly, including checking solutions in the original equation
- understood that an unknown to an even power is always positive and use this towards completing a proof.

Candidates who were awarded **Achievement with Excellence** commonly:

- used algebra skills to manipulate and solve equations involving complex numbers, including quotients, accurately and without unnecessary or confusing statements
- demonstrated problem-solving skills required to group real and imaginary terms, and applied the correct algebra to them, including completing the square and solving simultaneous equations with imaginary coefficients
- demonstrated understanding of what the modulus symbol required, and applied it successfully, including solving modulus problems resulting in surds
- communicated their thinking clearly and accurately about what they were doing, while completing multi-step problems
- provided precise, logical, and easy-to-follow working, reducing the chance of numerical or algebraic errors, and presented clear steps throughout their solutions
- showed careful thought in recognising when a chosen strategy was not likely to be successful, and then reassessed the chosen method
- displayed abstract thinking in solving unfamiliar problems, making links to the information provided in the question
- completed required proofs by making connections between real and imaginary parts

- explored problems where not all solutions would be valid and solved accordingly
- rationalised denominators and understood that an argument corresponds to where the real and imaginary parts of a complex number are equal.

Candidates who were awarded **Not Achieved** commonly:

- did not demonstrate basic algebra skills needed to simplify, solve, expand, and factorise
- displayed little understanding of the process for completing the square, theories relating to the discriminant, or quadratic formulae
- did not rationalise a denominator correctly
- were not familiar with the definitions of modulus and argument, and hence could not find the modulus or argument of a complex number
- did not manipulate complex numbers in either polar or rectangular form
- could not convert a complex number from one form to another (rectangular to polar or vice versa) before performing calculations with complex numbers
- did not demonstrate how to convert to polar form without the use of a calculator
- relied too heavily on the use of a graphics calculator
- showed no understanding of interpreting or using de Moivre’s theorem
- showed no understanding of, or how to interpret, complex numbers represented on an Argand diagram
- showed no understanding of or how to apply the factor or remainder theorem correctly
- made too many basic errors in simplifying and applying algebraic methods
- did not multiply surds together correctly, nor manipulate equations containing surds
- did not maintain consistency when solving equations
- did not attempt a sufficiently large proportion of the assessment.

Achievement standard 91578: Apply differentiation methods in solving problems

Assessment

This examination provided candidates with multiple opportunities to display their understanding of:

- derivatives of power, exponential, and logarithmic (base e only) functions
- derivatives of trigonometric (including the reciprocal trigonometric) functions
- optimisation
- equations of tangents and normal
- maxima, minima, and points of inflection
- related rates of change
- first and second derivatives of parametric functions
- chain, product, and quotient rules
- properties and features of piece-wise graphs (limits, differentiability, continuity, concavity).

Commentary

Candidates must ensure that they are fully confident in their own ability to be able to differentiate all various forms of the functions, including when the use of the chain, product, and quotient rules are necessary. Many of the candidates displayed the necessary understanding to be able to interpret the application using differentiation methods but were let down by errors in their methods of differentiation, including not recognising when product and quotient rules need to be implemented. Success can only be awarded if the applications of the differentiation rules are displayed accurately, error-free, and mathematically correct.

Candidates are reminded that evidence of clear working and methods must be shown at all levels of the examination. Answers provided without appropriate supporting working and full calculus justification are unlikely to be fully rewarded. Candidates should follow the guidance in those questions that stipulate "you do not need to simplify your answer", as many candidates continue to simplify their derivatives and make careless mistakes when doing so, resulting in them losing their grade in the question. This particularly applies to achievement level questions.

To solve the problems involving application of differentiation, candidates need to have sound algebra skills, as well as strong knowledge of the various differentiation methods, which come from regular and extensive practice from a wide variety of possibilities. Candidates need to be confident enough to solve quadratic equations, equations involving the use of exponentials and logarithms, and simple trigonometric equations, as well as manipulating rational expressions and trigonometric functions. Candidates should not be relying solely on their graphical calculators to solve such equations as the inclusion of unknown constants will generally eliminate this pathway. For all levels of success, a combination of both skills of differentiation and algebra are necessary and essential. Simply being able to differentiate various types of functions is not going to be enough to gain success in this standard. Background knowledge in coordinate geometry is also important, particularly for questions involving tangents, normals, gradients, and x-intercepts. The lack of brackets, especially when using the product, chain and quotient rules, continues to be a problem and ultimately leads to inaccurate solutions.

Modelling for optimisation problems continues to be a skill demonstrated by high-performing candidates. Candidates need a lot more practice at understanding how to deal with creating appropriate models within an optimisation problem.

Many candidates were not able to differentiate successfully the function provided in Question One (c), which involved a mixture of product, exponents, and trigonometry. This therefore blocked these candidates from being able to successfully apply their resulting expression. Many tried to use trigonometric identities to differentiate, but lacked the skills to apply these correctly. Others frequently omitted a factor of the differentiation, which led to an incorrect differentiation, so no credit could be given for this question part, even for candidates who subsequently managed to continue to find their solution.

Candidates struggled to find the correct second derivative for the parametric functions given in Question One (e) by omitting to multiply by $\frac{\mathrm{d}t}{\mathrm{d}x}$, having differentiated with respect to the parameter. This resulted in an answer supported by an invalid method.

Many were confused when the problem connected their differentiation and co-ordinate geometry knowledge. (Question One (d), Question Two (c)). It was evident from the errors seen that candidates were not sufficiently confident and experienced in this type of problem solving.

Candidates must communicate formally when finding stationary points and inflection points involving rational equations. Too many candidates remove the denominator before communicating that the gradient at these points is equal to zero. The marker needs to be shown every step of the process, with full justification given, whereas it is clear that candidates rely too much on their graphical calculator for their solutions. This was particularly the case with Question One (e), Question Two (e), Question Three (b), and Question Three (e).

When finding equations of tangents, candidates are expected to present answers in general or gradient-intercept form, rather than leaving equations with substituted values only. Final answers must also be fully evaluated to gain full credit in a question.

Candidates aiming for higher levels of achievement should be strongly discouraged from only attempting these questions. Many who have taken this approach have likely fallen short of the grade they are targeting due to errors made in these questions. All questions should be attempted to have required success in this examination.

Grade awarding

Candidates who were awarded **Achievement** commonly:

- used the chain rule, product rule, and quotient rule correctly in combination with power functions, trigonometric functions, parametric functions, and exponential and logarithmic (base e only) functions
- successfully wrote surd expressions into fractional index form, before differentiating
- solved quadratic, exponential, natural logarithmic, and power equations resulting from differentiating a function
- provided the conditions, using the derivative, for when a function is decreasing
- used differentiation to find the gradient of tangents of functions
- used a curriculum-appropriate model to solve an optimisation problem, and then correctly differentiated their model
- found the two components of a related rates of change problem and then correctly differentiated them
- calculated the *x*- and *y*-coordinates correctly for any stationary points and inflection points
- showed sufficient algebraic skills to simplify their answer having differentiated a function when required to do so
- confidently and knowledgeably used their calculators to evaluate functions and differentiated functions, including the correct use for trigonometric and exponential functions.

Candidates who were awarded **Achievement with Merit** commonly:

- solved equations involving quadratic functions correctly to find the coordinates of stationary points demonstrating confident algebraic skills
- demonstrated reliable and accurate algebraic skills and knowledge, with clear and full communication, when solving problems
- were able to use the product and quotient rules, and chain rule, confidently and correctly when differentiating functions
- communicated their solutions distinctly, with clear work, to demonstrate their understanding and methods, ensuring that the requirements of the question were fulfilled
- solved a related rates problem, involving the volume of a sphere
- used relational thinking to connect coordinate geometry and differentiation knowledge, and apply to gradients of points, tangents, and normals, as well as finding the equations of a tangents for parametric functions.
- demonstrated the procedure to set up models for optimisation questions, in terms of one variable, before correctly differentiating it
- found second differentials of parametric and trigonometric functions
- found the coordinates of the points on a curve for a given gradient
- found the coordinates of the point of intersection of two tangents to a curve
- found the coordinates of the point where a tangent to a curve crosses the *x*-axis.

Candidates who were awarded **Achievement with Excellence** commonly:

- demonstrated high-quality algebraic skills when solving problems resulting from differentiating, especially when simplifying expressions
- set up appropriate models and interpreted them correctly to solve the problem using differentiation
- manipulated surds and trigonometric functions correctly, resulting from differentiating and evaluating
- confidently found and used the second derivative of parametric and trigonometric functions when solving problems
- demonstrated abstract thinking while applying differentiation methods in solving unfamiliar problems, utilising precise and accurate algebraic methods
- linked the first and second derivative of parametric functions to find the stationary points of inflection on a curve
- evaluated the first and second derivative of a trigonometric function and used this to find the radius of curvature on a curve
- formed an area function for a rectangle and found the optimal solution that maximises the area and used this solution to find the base length of the rectangle
- formed, interpreted and solved problems related to maximums, inflections, and stationary points.

Candidates who were awarded **Not Achieved** commonly:

- did not apply the chain rule, product rule or quotient rule in combination with power, trigonometric functions, parametric functions, exponential, and logarithmic (base e only) functions correctly
- did not recognise when it was necessary to apply the product and quotient rules for differentiation
- did not use the product and quotient formulae correctly
- did not solve a resulting equation algebraically, even though they may have found the correct derivative
- did not correctly factorise and solve quadratic expressions, even if they had found the correct derivative
- made careless errors when solving equations, and were unable to self-check their solutions
- demonstrated poor algebraic skills, such as inaccurate expanding, factorising, incorrectly simplifying derivatives, substitution, cancelling of factors, and manipulating algebraic fractions
- did not communicate that gradients are zero at stationary points by setting full derivatives equal to zero
- did not recognise features of gradient, continuity, and limits from a piece-wise graph
- did not actually answer the full requirements of the question.

Achievement standard 91579: Apply integration methods in solving problems

Assessment

This examination provided candidates with multiple opportunities to display their understanding of:

- integrating power, polynomial, exponential (base e only), trigonometric, and rational functions
- reverse chain rule, trigonometric formulae
- rates of change problems
- areas under or between graphs of functions, by integration
- finding areas using numerical methods, e.g. Simpson's rule or trapezium rule

- differential equations of the forms $y' = f(x)$ or $y'' = f(x)$ for the above functions or situations where the variables are separable (e.g. $y' = ky$) in applications such as growth and decay, inflation, Newton's law of cooling, and similar situations.

Commentary

Candidates should be strongly discouraged from only attempting the "Excellence" part of each question. It was very common for candidates who did this to make an error on one or more of these and ultimately under-perform on their intended and desired target. Conversely, many candidates enhanced their overall final grade by attempting all parts of all questions, thereby gaining grade u or grade r rewards on the merit or excellence question parts.

Candidates should be encouraged to use the formal and correct terminology throughout the solution to a problem, using a step-by-step strategy, in order to avoid careless errors and to guide the candidate through each stage of the necessary working and calculations, including the accurate use of the constant of integration. Within the teaching and learning programme it needs to be stressed that candidates must correctly use the constant of integration. The constant of integration should not be omitted and, additionally, its value need not necessarily be zero. Within the formal working, while solving a differential equation, care must be taken to use the constant of integration consistently throughout a solution.

Candidates are reminded that evidence of the correct use of integration methods must be clearly demonstrated. Answers from a graphical calculator are unlikely to attain higher than an "Achieved" grade, at best.

Candidates are reminded that correct and formal notation is expected, which includes the use of brackets where necessary. If algebraic skills are not sufficiently strong, this will have an impact on the level of success on this integration methods achievement standard.

Candidates need to take care in avoiding manipulation errors, e.g. numeric errors, sign errors, transfer errors. These could be because of the necessity of working under pressure but also could be because of a lack of a deeper understanding of the necessary mathematical concepts.

Candidates frequently omitted a factor of the integration, which led to an incorrect integration, so no credit could be given for this question part, even for candidates who managed to arrive at the correct solution.

In Question Two (d), a kinematics question, a significant number of candidates assumed that as $s = 0$ when $t = 0$, then the constant of integration, c, would also be $c = 0$. The displacement equation was an exponential function,n and consequently this was not the case.

In Question Two (c), Question Two (d), and Question Three (d), a significant number of candidates integrated and substituted correctly, but then were not successful in completing the problem because of algebraic errors. This prevented these candidates from accessing the necessary points range for merit attainment.

Grade awarding

Candidates who were awarded **Achievement** commonly:

- manipulated expressions into forms that could be integrated
- successfully used algebra to solve simple equations
- successfully wrote surd expressions in exponent form
- integrated exponential and trigonometric functions successfully
- successfully found the constant of integration given the supplied information
- successfully substituted in the limits of a definite integral to find the required area
- successfully used algebra to solve an unknown after an integration

- successfully applied the trapezium rule
- successfully integrated a function of the type $(ax + b)^n$, i.e. could use the 'reverse chain rule'.

Candidates who were awarded **Achievement with Merit** commonly:

- demonstrated the use the limits of integration correctly
- demonstrated how to find the area under curves
- used algebra appropriately and reliably
- successfully integrated a function of the type $(ax + b)^n$, i.e. could use the 'reverse chain rule'
- successfully turned a trigonometric product into a trigonometric sum in order to integrate
- could successfully separate variables
- understood the relationship between acceleration, velocity and displacement
- Interpreted given information regarding velocity and displacement correctly
- recognised that $\frac{x^2+6}{x^4} = \frac{1}{x^2} + \frac{6}{x^4}$
- could manipulate $\frac{-1}{2p} - \frac{2}{(2p)^3} + \frac{1}{p} + \frac{2}{p^3} = \frac{9}{4}$ into $9p^3 - 2p^2 - 7 = 0$, showing correct algebraic working to complete a proof used brackets carefully and reliably.

Candidates who were awarded **Achievement with Excellence** commonly:

- demonstrated the ability to solve problems involving areas and differential equations using the correct notation
- demonstrated the ability to manipulate expressions including long division, solve equations by splitting the variables, and using trigonometric identities to a form that could be integrated.

Candidates who were awarded **Not Achieved** commonly:

- did not recognise the constant of integration (+ *c*) or assumed its value was zero
- did not recognise the different methods to integrate a rational function, i.e. reverse of the 'chain rule', logarithm function, separate, and simplify
- were unable to integrate functions that required algebraic manipulation first
- did not attempt all questions
- integrated any function with f(x) in the denominator to ln(f(x))
- did not recognise the value of h in the Trapezium rule
- did not 'separate the variables' successfully
- misinterpreted initial value information in kinematics
- did not recognise that $\sqrt{x}$ is $x^{\frac{1}{2}}$
- did not recognise that $\frac{1}{\sqrt{x}}$ is $x^{\frac{-1}{2}}$
- did not use algebra correctly to solve simple equations
- had only attempted the excellence questions.