Assessment Schedule – 2011

Mathematics and Statistics: Apply geometric reasoning in solving problems (91031)

Evidence Statement

Achievement	Achievement with Merit	Achievement with Excellence
Apply geometric reasoning in solving problems will involve: • using a range of methods when solving problems • demonstrating knowledge of geometrical concepts and terms, and • communicating solutions which would usually require only one or two steps.	*Relational thinking* will involve one or more of: • selecting and carrying out a logical sequence of steps • connecting different concepts and representations • demonstrating understanding of concepts • forming and using a model, • relating findings to a context • communicating thinking using appropriate mathematical statements.	*Extended abstract thinking* will involve one or more of: • devising a strategy to investigate or solve a problem • identifying relevant concepts in context • developing a chain of logical reasoning, or proof • forming a generalisation, • using correct mathematical statements, • communicating mathematical insight.

Sufficiency for all questions :

N0 – no response, no relevant evidence
N1 – one step on two questions
N2 – 1u

A3 – 2u
A4 – 3u

M5 – 2r
M6 – 3r

E7 – 1t
E8 – 2t

(1u, 1r = A3)

(2u, 1r = A4)

Question	Evidence	Achievement (u)	Merit (r)	Excellence (t)
		Apply geometric reasoning in solving problems.	*Apply geometric reasoning, using relational thinking, in solving problems.*	*Apply geometric reasoning, using extended abstract thinking, in solving problems.*
ONE (a)(i)	XYB = 62° (corresponding angles) YBC = XYB = 62° (alt angles) OR opposite angles in a parallelogram are equal OR equivalent.	Finding angle YBC correctly.	Finding angle YBC correctly, giving a coherent explanation of the geometric reasoning.	
(a)(ii)	XYB = 62° (corresponding angles) BYC = 54° (sum of angles of triangle) If XB is parallel to YC YBX = 54° (alternate angles) Therefore BXY would be 64° so the triangle would not be isosceles. *So if the triangle is isosceles, XB and YC cannot be parallel.*		Clearly and logically explaining, with geometric reasons which outline TWO steps of logic.	Clearly and logically explaining, with geometric reasons which outline all THREE steps of logic.
(b)(i)	Obtuse COA = $2 \times 72° = 144°$ (Angle at centre) Reflex COA, $x = 360° - 144° = 216°$ (angles at a point)	Finding angle x correctly.	Finding angle x correctly, giving a coherent explanation of the geometric reasoning.	
(b)(ii)	$y = 360° - (216° + 72° + 38°) = 34°$ (angles in a quad)	Finding angle DCO correctly.	Finding angle DCO correctly, giving a coherent explanation of the geometric reasoning. OR 106 for consistency from b(i) using 144	
(b)(iii)	Draw BO and DO DOB = $2 \times$ DCB (angle at centre) Reflex DOB = $2 \times$ BAD (ditto) DOB + reflex DOB = 360° (angle at a point) So 2DCB + 2BAD = 360° hence DCB + BAD = 180°.	At least TWO steps towards a proof. OR any identification that angles at centre twice angles at circumference	Coherent explanation of reasoning.	Giving a clear, generalised explanation of the relationship, using correct geometric reasoning.

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		Apply geometric reasoning in solving problems.	*Apply geometric reasoning, using relational thinking, in solving problems.*	*Apply geometric reasoning, using extended abstract thinking, in solving problems.*
TWO (a)(i)	PT = $\sqrt{90^2 - 70^2} = 56.6$ cm (3sf)	Finding PT correctly.		
(ii)	$\sin^{-1}(\frac{70}{90}) = 51.05755873$ So PTO is 51.1° to 1dp	Finding angle PTO correctly.		
(iii)	Similar triangles: $\begin{pmatrix} OQR \\ OPT \end{pmatrix}$ since they are both right-angled and they share angle TOP so $\frac{OR}{OT} = \frac{OQ}{OP}$ $\frac{OR}{90} = \frac{30}{70}$ OR = 38.6 cm (3sf) OR: Angle TOP = 180 – (90 + 51.1) = 38.9 (angles in a triangle) So $\frac{30}{OR} = \cos(38.9)$.. OR = 38.4 cm (3sf)	Finding angle TOP correctly. OR Using trigonometry or similar triangles to find a length correctly, whether relevant or not.	Deducing the length of OR correctly but not expressing the process in a clear, logical manner. OR Correct answer only.	Deducing, in a clear, logical manner, the correct length.
(b)(i)	• TNP is an equilateral or isosceles triangle (since all the sides are the same length). • AP is the line of symmetry in triangle TNP or symmetrical. • TAP must be 90° (angles on line). OR equivalent.	Giving an explanation for the result, which is not complete, but has TWO of the THREE points clearly made. OR Answer and one reason.	Giving a clear, complete and geometrically correct explanation with correct answer.	
(ii)	$AP = \sqrt{40^2 - 20^2} = 20\tan(60)$ = 34.6 cm (3sf)	Finding AP correctly by either method.		
(iii)	In triangle ONA, OA = $\sqrt{90^2 - 20^2} = 87.7$ cm (4sf) In triangle OAP, angle to the ground is angle OAP = $\cos^{-1}(\frac{34.6}{87.7}) = 66.76°$ (4sf) OR In triangle OTP OP=$\sqrt{(90^2-40^2)}$ = 80.62 so Angle OAP =$\tan^{-1}$(80.62/34.6) = 66.77	One correct trig ratio or Pythagoras.	Making progress towards answering the question by working consistently in either plane ONT or plane OAP, but not gaining the final correct angle.	Deducing the correct size of the angle between the planes, giving clear explanation of the method used to do so.

Question	Evidence	Achievement (u)	Merit (r)	Excellence (t)
		Apply geometric reasoning in solving problems.	*Apply geometric reasoning, using relational thinking, in solving problems.*	*Apply geometric reasoning, using extended abstract thinking, in solving problems.*
THREE (a)(i)	RQP = 70° (angles on line) RPQ = 70° (base angles isos) PRQ = 40° (angles in triangle)	Finding angle PRQ correctly. (Correct answer only.)	Finding angle PRQ correctly, giving a coherent explanation of the geometric reasoning with three steps.	
(ii)	RQP = 180° – x (angles on line) RPQ = 180° – x (base angles isos) PRQ = 180° – 2(180° – x) = 2x – 180° (angles in triangle) QRT = 180 – x (co-int angles) PRT = PRQ + TRQ = (2x – 180°) + (180° – x) = x = RQS (or equivalent)	Giving TWO coherent steps towards to a proof. OR finding base angles correctly.	Coherent explanation of reasoning. OR finding base angles correctly and one more step with correct geometric reasoning.	Giving a clear, generalised, logical explanation of the relationship between x and PRT, using correct geometric reasoning.
(b)	EFG = 98° (opp angles cyc quad) HFJ = 98° (vert opp angles)	Finding angle HFJ correctly.	Finding angle HFJ correctly, giving a coherent explanation of the geometric reasoning.	
(c)(i)	NAQ = 180 – x (co-int angles) NZQ = 180 – (180 – x) = x (opp angles cyclic quad)	Finding the correct expression for angle NZQ, without a clearly explained sequence of steps with geometric justification.	Finding an expression for angle NZQ coherently, using a clearly explained sequence of steps with geometric justification.	
(ii)	Expaining that a cyclic parallelogram will need to have 90° in each corner since it is a cyclic trapezium, so the relationships in part (i) must hold. In addition, adjacent corners are co-interior so must also add up to 180. This means that x and 180° – x must be equal, so x = 90°. Hence a cyclic parallelogram must be a rectangle and have right angles in each corner. Opp. angles of a cyclic quad. total 180°. Opp. angles in a parallelogram are equal. Cointerior angles of a parallelogram total 180°.	Explaining that a cyclic parallelogram must have the geometric properties of a cyclic trapezium, but not being able to advance beyond this. OR just 90°. OR two correct reasons.	90° and one correct reason.	Explaining, using clear, logical geometric reasoning, that a cyclic parallelogram must be a rectangle. Ie, 90° and two correct reasons.

Judgement Statement

	Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
Score range	0 – 6	7 – 13	14 – 19	20 – 24