Assessment Schedule – 2015

Calculus: Apply integration methods in solving problems (91579)

Evidence

Q1	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$\frac{2}{3}x^{\frac{3}{2}} + 3\sin 2x + c$	Correct integral.		
(b)	$y = \int \frac{2}{x}\mathrm{d}x = 2\ln x + c$ $x = 1, y = 3 \Rightarrow 3 = 2\ln 1 + c$ $c = 3$ $y = 2\ln x + 3$	Correct solution with correct integral.		
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x} = \frac{\mathrm{e}^{2x}}{4y} \Rightarrow \int 4y\,\mathrm{d}y = \int \mathrm{e}^{2x}\mathrm{d}x$ $2y^2 = \frac{1}{2}\mathrm{e}^{2x} + c$ $x = 0, y = 4 \Rightarrow 32 = \frac{1}{2} + c$ $\Rightarrow c = \frac{63}{2}$ $\therefore 2y^2 = \frac{1}{2}\mathrm{e}^{2x} + \frac{63}{2}$ $x = 2 \Rightarrow 2y^2 = \frac{1}{2}\mathrm{e}^4 + \frac{63}{2}$ $= 58.8$ $y = \sqrt{29.4} = 5.42$	Correct integral.	Correct integral and correct solution.	
(d)	$\int_2^5 \frac{5x-3}{x+3}\mathrm{d}x = \int_2^5 \left(5 - \frac{18}{x+3}\right)\mathrm{d}x$ $= \left[5x - 18\ln(x+3) + c\right]_2^5$ $= (25 - 18\ln 8) - (10 - 18\ln 5)$ $= 15 + 18(\ln 5 - \ln 8)$ $= 15 + 18\ln\left(\frac{5}{8}\right)$ $= 6.54$	Correct integral.	Correct solution with correct integral.	

Q1	Expected Coverage	Achievement u	Merit r	Excellence t
(e)	$\text{Vol} = \pi\int_0^{\frac{\pi}{2}} (\cos x)^2 \mathrm{d}x$ $= \pi\int_0^{\frac{\pi}{2}} (\cos^2 x)\mathrm{d}x$ $= \frac{\pi}{2}\int_0^{\frac{\pi}{2}} (\cos 2x + 1)\mathrm{d}x$ $= \frac{\pi}{2}\left[\frac{\sin 2x}{2} + x\right]_0^{\frac{\pi}{2}}$ $= \frac{\pi}{2}\left[\left(\frac{\sin\pi}{2} - \frac{\pi}{2}\right) - (\sin 0 - 0)\right]$ $= \frac{\pi}{2} \times \frac{\pi}{2}$ $= \frac{\pi^2}{4}$ $(= 2.467)$		Correct integral (3rd line).	Correct solution with correct integral.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	ONE correct integration.	2u	3u	1r	2r	1t with minor errors ignored.	1t

Q2	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$\int (3-\frac{5}{x^2})\mathrm{d}x = \int (3-5x^{-2})\mathrm{d}x$ $= 3x+5x^{-1}+c$	Correct integral.		
(b)	$\text{Area} \approx \frac{0.25}{2}\left[0.3+1.1+2(0.7+1.65+1.9+2.35+1.7)\right]$ $= \frac{1}{8}\times 18$ $= 2.25$	Correct solution.		
(c)	$\frac{\mathrm{d}v}{\mathrm{d}t} = \frac{50t^2-80\sqrt{t}}{5\sqrt{t}} = 10t^{\frac{3}{2}}-16$ $v = \int\left(10t^{\frac{3}{2}}-16\right)\mathrm{d}t$ $= 4t^{\frac{5}{2}}-16t+c$ $t=0, v=6 \Rightarrow c=6$ $\therefore v = 4t^{\frac{5}{2}}-16t+6$ $t=4 \Rightarrow v = 70 \text{ m s}^{-1}$	Correct integral.	Correct integral and correct solution. Units not required.	
(d)(i) (ii)	$\frac{\mathrm{d}P}{\mathrm{d}t} = kP$ $\int \frac{1}{P}\mathrm{d}P = \int \mathrm{k}\,\mathrm{d}t$ $\ln P = \mathrm{k}t+c$ $P = \mathrm{e}^{\mathrm{k}t+c}$ $P = A\mathrm{e}^{\mathrm{k}t}$ $t=0, P=12\,000 \Rightarrow A = 12\,000 \Rightarrow P = 12\,000\mathrm{e}^{\mathrm{k}t}$ $t=10, P=16\,000 \Rightarrow 16\,000 = 12\,000\mathrm{e}^{10\mathrm{k}}$ $\frac{4}{3} = \mathrm{e}^{10\mathrm{k}} \Rightarrow 10k = \ln\left(\frac{4}{3}\right)$ $k = 0.0288$ $\therefore P = 12\,000\mathrm{e}^{0.0288t}$ $t=25 \Rightarrow P = 12\,000\mathrm{e}^{0.0288\times 25}$ $= 24\,653$	Correct differential equation in (i) and correct solution to DE (line 3) in (ii).	Correct DE in (i) and correct solution to DE (line 3) in (ii). Plus correct solution. Final value may vary depending on rounding of k.	

(e)	$\frac{2}{x-1}=x$ $x^2-x-2=0$ $(x-2)(x+1)=0$ $x=2$ or $x=-1$ The point we want is $x=2$. $\int_2^k \left(\frac{2}{x-1}\right)\mathrm{d}x=4$ $\left[2\ln(x-1)\right]_2^k=4$ $2\ln(k-1)-2\ln(1)=4$ $2\ln(k-1)=4$ $\ln(k-1)=2$ $k-1=\mathrm{e}^2$ $k=\mathrm{e}^2+1$ $=8.39$	Correct line 7, but with incorrect or no limits.	Correct integration with correct limits substituted.	Correct solution with correct integral. Accept e^2+1.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	ONE correct integration.	2u	3u	1r	2r	1t with minor errors ignored.	1t

Q3	Expected Coverage	Achievement u	Merit r	Excellence t
(a)	$\frac{x^3}{3}+4x^2+16x+2e^{4x}+c$	Correct integral.		
(b)	1	Correct solution.		
(c)	$\text{Area }=\int_0^{\frac{\pi}{k}}\sin kx\,\mathrm{d}x$ $=\left[\frac{-\cos kx}{k}+c\right]_0^{\frac{\pi}{k}}$ $=\left[\frac{-\cos\pi}{k}\right]-\left[\frac{-\cos 0}{k}\right]$ $=\frac{1}{k}-\frac{-1}{k}$ $=\frac{2}{k}$	Correct integral.	Correct integral and correct solution.	
(d)	$-x^2+2=x^3-x^2-kx+2$ $x^3-kx=0$ $x\left(x^2-k\right)=0$ $x=\pm\sqrt{k}$ $\text{Area A}=\int_{-\sqrt{k}}^{0}((x^3-x^2-kx+2)-(-x^2+2))\mathrm{d}x$ $=\int_{-\sqrt{k}}^{0}\left(x^3-kx\right)\mathrm{d}x$ $=\left[\frac{x^4}{4}-\frac{kx^2}{2}\right]_{-\sqrt{k}}^{0}$ $=0-\left(\frac{k^2}{4}-\frac{k^2}{2}\right)$ $=\frac{k^2}{4}$ $\text{Area B}=\int_0^{\sqrt{k}}((-x^2+2)-(x^3-x^2-kx+2))\mathrm{d}x$ $=\int_0^{\sqrt{k}}\left(kx-x^3\right)\mathrm{d}x$ $=\left[\frac{kx^2}{2}-\frac{x^4}{4}\right]_0^{\sqrt{k}}$ $=\left(\frac{k^2}{2}-\frac{k^2}{4}\right)$ $=\frac{k^2}{4}$ Therefore both regions have the same area.	Correct integration – either region.	Correct solution with correct integrations.	

(e)	$a = B\left(e^{kt}\right)^2$ $= Be^{2kt}$ $v = \int Be^{2kt}$ $= \frac{B}{2k}e^{2kt} + c$ At $t = 0$, $v = 0$ $\Rightarrow 0 = \frac{B}{2k} + c$ $c = \frac{-B}{2k}$ $v = \frac{B}{2k}e^{2kt} - \frac{B}{2k}$ $= \frac{B}{2k}\left(e^{2kt} - 1\right)$ $v = v_0$ $v_0 = \frac{B}{2k}\left(e^{2kt} - 1\right)$ $\frac{2kv_0}{B} + 1 = e^{2kt}$ $\frac{2kv_0 + B}{B} = e^{2kt}$ $\ln\left(\frac{2kv_0 + B}{B}\right) = 2kt$ $t = \frac{1}{2k}\ln\left(\frac{2kv_0 + B}{B}\right)$		Correct integration (4th line).	Correct solution.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	ONE correct integration.	2u	3u	1r	2r	1t with minor errors ignored.	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 13	14 – 20	21 – 24