Assessment Schedule – 2017

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

Q 1	Evidence	Achievement	Merit	Excellence
(a)	$2\tan 2x + c$	Correct solution.		
(b)	$\text{Area} = \int_1^4 \left(x + x^{\frac{-1}{2}} \right) dx$ $= \left[\frac{x^2}{2} + \frac{x^{\frac{1}{2}}}{\frac{1}{2}} \right]_1^4$ $= \left[\frac{x^2}{2} + 2\sqrt{x} \right]_1^4$ $= (8+4) - \left(\frac{1}{2} + 2 \right)$ $= 9.5$	Correct solution with correct integration.		
(c)	$v(t) = \int 1.2t^{\frac{1}{2}} dt$ $= 0.8t^{\frac{3}{2}} + c$ $t = 4,\ v = 7$ $7 = 0.8(4)^{\frac{3}{2}} + c$ $7 = 6.4 + c$ $c = 0.6$ $v(t) = 0.8t^{\frac{3}{2}} + 0.6$ $d(t) = \int 0.8t^{\frac{3}{2}} + 0.6$ $= 0.32t^{\frac{5}{2}} + 0.6t + c$ $d(9) = 0.32(9)^{\frac{5}{2}} + 0.6 \times 9 = 83.16 \text{ m}$	Correct expression for *v(t)* with constant *c* calculated	Correct solution with correct integrals. Units not required. Stating $c = 0$ not required.	
(d)	$\int_0^k 3e^{2x} dx = 4$ $\frac{3}{2}\left[e^{2x} \right]_0^k = 4$ $e^{2k} - e^0 = \frac{8}{3}$ $e^{2k} = \frac{11}{3}$ $k = \frac{1}{2}\ln\left(\frac{11}{3} \right) = 0.6496$	Correct integration.	Correct solution with correct integration.	

(e)	$\cos 2x = 1 - 2\sin^2 x$ $\therefore \sin^2 x = \frac{1}{2}(1 - \cos 2x)$ Mean value $= \dfrac{\int_0^{\pi} \sin^2 x \, dx}{\pi - 0}$ $= \dfrac{\frac{1}{2}\int_0^{\pi}(1 - \cos 2x)\,dx}{\pi}$ $= \dfrac{\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi}}{2\pi}$ $= \dfrac{\left(\pi - \frac{1}{2}\sin 2\pi\right) - (0 - 0)}{2\pi}$ $= \frac{1}{2}$		Correct integration of $\sin^2 x$ expression.	Correct solution with correct integration of $\sin^2 x$.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Evidence	Achievement	Merit	Excellence

(a)	$3\ln(2x-1)+c$	Correct solution.		
(b)	$\frac{(2x-5)^5}{10}+c$ OR $\frac{16x^5}{5}-40x^4+200x^3-500x^2+625x+c$	Correct solution.		
(c)	$\text{Area}=\int_2^{14}(-x+14)dx-\int_2^5(-x^2+3x+10)dx$ $=\left[\frac{-x^2}{2}+14x\right]_2^{14}-\left[\frac{-x^3}{3}+\frac{3x^2}{2}+10x\right]_2^5$ $=72-22.5$ $=49.5$ OR $\text{Area}=\frac{1}{2}\times12\times12-\int_2^5(-x^2+3x+10)\mathrm{d}x$ $=72-\left[\frac{-x^3}{3}+\frac{3x^2}{2}+10x\right]_2^5$ $=72-[45.833-23.333]$ $=72-22.5$ $=49.5$	Correct integration of both expressions. OR Correct integration.	Correct solution with correct integration.	
(d)	$\text{Area }=\int_0^{\frac{\pi}{4}}\sin3x\cos2x\,\mathrm{d}x$ $=\frac{1}{2}\int_0^{\frac{\pi}{4}}(\sin5x+\sin x)\mathrm{d}x$ $=\frac{1}{2}\left[\frac{-\cos5x}{5}-\cos x\right]_0^{\frac{\pi}{4}}$ $=\frac{1}{2}\left[\left(\frac{-1}{5}\cos\frac{5\pi}{4}-\cos\left(\frac{\pi}{4}\right)\right)-\left(\frac{-1}{5}\cos0-\cos0\right)\right]$ $=\frac{1}{2}[0.1414-0.7071+0.2+1]$ $=0.3172$	Correct integration.	Correct solution with correct integration.	

(e)	$v = \int \frac{20\ln t}{t}\,\mathrm{d}t$ $= 20\int \frac{1}{t}\ln t\,\mathrm{d}t$ $= 20 \times \frac{(\ln t)^2}{2} + c$ $= 10(\ln t)^2 + c$ $t = 4,\ v = 12$ $12 = 10(\ln 4)^2 + c$ $c = 12 - 10(\ln 4)^2 = -7.218$ $v = 10(\ln t)^2 - 7.218$ $t = 10$ $v = 10(\ln 10)^2 - 7.218$ $v = 45.8\ \mathrm{m\ s^{-1}}$		Correct integration.	Correct solution with correct integration.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	Evidence	Achievement	Merit	Excellence
(a)	$-3x^{-3}+2e^{4x}+c$	Correct solution.		
(b)	82 m^2	Correct solution (units not required).		
(c)	Area $=\int_1^{11}\frac{15x-15}{x+2}dx$ $=15\int_1^{11}\left(1-\frac{3}{x+2}\right)dx$ $=15\left[x-3\ln(x+2)\right]_1^{11}$ $=15\left[(11-3\ln 13)-(1-3\ln 3)\right]$ $=15\left[10+3\ln\left(\frac{3}{13}\right)\right]$ $=84.015\text{ m}^2$ If use substitution this is the integrated expression: $15(x+2)-45\ln(x+2)$	Correct integration.	Correct solution with correct integration. Units not required.	
(d)	$\int\frac{1}{y}dy=\int\frac{1}{\sqrt{x}}dx$ $\ln y=2x^{\frac{1}{2}}+c$ $\ln y=2\sqrt{x}+c$ $x=4,\ y=1$ $0=4+c$ $c=-4$ $\ln y=2\sqrt{x}-4$ $y=e^{2\sqrt{x}-4}$ or $y=0.0183e^{2\sqrt{x}}$	Correct integration.	Correct solution with correct integration. Accept log form.	

(e)	$\frac{dy}{dt} = k\cos 0.5te^{\sin 0.5t}$ $y = 2ke^{\sin 0.5t} + c$ $t = 0, y = 8$ $8 = 2ke^0 + c$ $8 = 2k + c$ $t = 2, y = 12$ $12 = 2ke^{\sin 1} + c$ $12 = 2k \times 2.34 + c$ $12 = 4.64k + c$ $\therefore 4 = 2.64k$ $k = \frac{4}{2.64} = 1.52$ $c = 8 - (2 \times 1.52) = 4.96$ $\therefore y = 3.04e^{\sin 0.5t} + 4.96$ $t = 5 \Rightarrow y = 3.04e^{\sin 2.5} + 4.96$ $y = 10.49$		Correct integration.	Correct solution with correct integration.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of differentiation techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 12	13 – 18	19 – 24