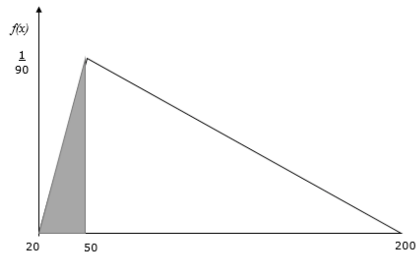
Assessment Schedule – 2017

Mathematics and Statistics (Statistics): Apply probability distributions in solving problems (91586)

Evidence Statement

One	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	 $P(X < 50) = 30 \times 0.5 \times \frac{1}{90} = 0.167$ Around 16.7% of showers use less than 50 litres of water per shower.	Correct probability calculated.		
(ii)	$P(X > 40) = 1 - P(X \leq 40)$ $= 1 - \left(\frac{1}{2} \times 20 \times \frac{1}{135}\right) = 0.926$ So around 92.6% of showers use more than 40 litres of water	$P(X \leq 40)$ $= 0.07407$	$P(X > 40)$ correctly calculated.	
(b)(i)	Binomial distribution $n = 10, p = 0.15$ $P(X \leq 4) = 0.990$	Correct probability calculated for (b)(i).	Correct probability calculated for (b)(i). AND Model identified as binomial and justified with two conditions linked to the context.	
(ii)	Binomial because: • fixed number of trials (10 trips) • fixed probability of success (15% of trips classified as "not accurate") • only two outcomes (not accurate or accurate) • independent events (time of randomly selected trips).			
(iii)	• The normal distribution model would provide negative absolute differences (not possible) [knowledge of unboundedness]. • A large proportion of trips would be modelled to have absolute differences less than 0 minutes [knowledge of mean / standard deviation]. • Absolute differences would be positively skewed (0 or higher, no upper bound) but model is symmetric [knowledge of symmetry]. • Under the normal distribution model given, $P(X \leq 5) = 0.704 \neq 0.87$ [comparing model probability to experimental probability]. *Accept other valid reasons.*	Describes one reason why the use of the normal distribution model is inappropriate, but does not link the feature with the context or vice versa, e.g. can't have negative absolute differences.	Describes one reason why the use of the normal distribution model is inappropriate and links this feature to the context.	Discusses two reasons why the use of the normal distribution model is inappropriate and clearly links these features to the context.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Reasonable start / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t (with minor omission or error)	1 of t

Two	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	$E(X) = 2.2$ $E(X^2) = 0^2 \times 0.11 + 1^2 \times 0.21 + 2^2 \times 0.24 + 3^2 \times 0.25 + 4^2 \times 0.19 = 6.46$ $VAR(X) = 6.46 - 2.2^2 = 1.62$	Correct calculation of VAR(X) in (a)(i). OR Correct explanation given as to why X and Y are not independent consistently using incorrect VAR(X) from (a)(i).	Correct calculation of VAR(X) in (a)(i). AND Correct explanation given as to why X and Y are not independent.	
(ii)	$VAR(X) = 1.62$ $VAR(Y) = 1.5376$ $VAR(X + Y) = 5.5696$ If X and Y are independent, $VAR(X + Y) = VAR(X) + VAR(Y)$ $1.5376 + 1.62 \neq 5.5696$ $3.1576 \neq 5.5696$ Therefore X and Y are not independent.			
(b)(i)	Normal distribution, $\mu = 17.8°$, $\sigma = 2.1°$ Central 95% = (13.68°, 21.92°) *Using graphics calculator.* Would expect 95% of NZ living rooms during winter months to have average temperatures between 13.68° and 21.92°.	Correct central 95% calculated in (b)(i). OR One factor described and linked to potential impact on temperatures modelled in (b)(ii).	Correct central 95% calculated in (b)(i). AND One factor described and linked to potential impact on temperatures modelled in (b)(ii).	
(ii)	Possible factors and potential impact on temperature: • insulation → houses that are insulated could be warmer • region of NZ → houses in South Island could be colder • age of house → houses that are older could be colder. *Accept other reasonable factors with link to model for temperature.*			
(iii)	Normal distribution, $\mu = 17.8°$, $\sigma = 2.1°$ $P(X < 16) = 0.1957$ Binomial distribution, n = 5, p = 0.1957 $P(X \geq 4) = 1 - P(X \leq 3) = 1 - 0.9938 = 0.0062$ Yes, finding four or more houses out of five with an average temperature of the living room below 16°C is unlikely (smaller than 5% chance) under the probability distribution model used for temperatures.	$P(X < 16)$ correctly calculated using normal distribution model. OR Uses Binomial, p = 0.8, n = 5 $P(X \geq 4)$ $= 1 - 0.26272$ $= 0.73728$	$P(X = 4)$ or $P(X \geq 4)$ correctly calculated using either a binomial distribution model or the multiplicative principle.	Correctly reasons that this observation would be unlikely under the model, supporting with a correct probability calculation.

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Three	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	Poisson distribution, $\lambda = 4.7$ $P(X \leq 4) = 0.495$	Correct probability calculated in (a)(i). OR Reason provided is linked to requirement for constant rate in (a)(ii).	Correct probability calculated in (a)(i). AND Reason provided is linked to requirement for constant rate in (a)(ii).	
(ii)	People may not flush the toilet at the same rate during different times of the day e.g. night time (when they might sleep for eight hours), so not constant rate for number of toilet flushes for any 4-hour period.			
(b)(i)	$E(X) = 1 \times 0.01 + 2 \times 0.1 + 3 \times 0.17 + 4 \times 0.26 + 5 \times 0.14 + 6 \times 0.12 + 7 \times 0.11 + 8 \times 0.05 + 9 \times 0.01 + 10 \times 0.03 = 4.74$ *Note: do not accept use of inverse Poisson distribution calculations to determine the mean.*	Correct mean calculated in (b)(i).	Correct mean calculated. AND Correct probability calculated and conclusion made.	
(ii)	Using the data, $P(X \geq 4) = 0.72$. Over half (most) of the people in the study flushed the toilet at least four times during a 24-hour period.	Correct probability calculated in (b)(ii).		
(iii)	Possible reasons why a Poisson model with $\lambda = 4.7$ is not a good model for the number of toilet flushes per 24-hour period: • You would expect a person to use the toilet (and so flush the toilet) more than once a day, but the model predicts 5.2% of people will use the toilet no more than once a day, around five times the proportion in the study (1%). • The model predicts roughly 19% of people will flush the toilet four times a day, but 26% of people in the study flushed the toilet four times in one day (8 percentage points higher). • Sometimes a person needs to flush more than once to complete the job so the event of each toilet flush might not be independent. *Accept other valid reasons based on modelling the number of toilet flushes per person per 24-hour period.*	One reason discussed but not fully linked to using context / experimental data OR model.	One reason discussed fully (using context / experimental data AND model).	Two reasons discussed fully (using context / experimental data AND model).

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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 18	19 – 24