Assessment Schedule – 2019

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

Q1	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$2x+4x^{\frac{1}{2}}+c$	Correct solution.		
(b)	6.8	Correct solution.		
(c)	$\int_0^{\frac{\pi}{12}} \cos 4x . \cos 2x \, dx$ $=\frac{1}{2}\int_0^{\frac{\pi}{12}}(\cos 6x+\cos 2x)\,dx$ $=\frac{1}{2}\left[\frac{\sin 6x}{6}+\frac{\sin 2x}{2}\right]_0^{\frac{\pi}{12}}$ $=\frac{1}{2}\left[\left(\frac{\sin\frac{\pi}{2}}{6}+\frac{\sin\frac{\pi}{6}}{2}\right)-(0+0)\right]$ $=\frac{1}{2}\left(\frac{1}{6}+\frac{1}{4}\right)$ $=\frac{5}{24}$	Correct integration.	Correct solution with correct integration.	
(d)	$\int_0^{16}\frac{6}{\sqrt{3x+1}}\,dx=\left[4(3x+1)^{\frac{1}{2}}\right]_0^{16}$ $=4\times\sqrt{49}-4\times\sqrt{1}$ $=24$ $\int_0^{5}\frac{6}{\sqrt{3x+1}}\,dx=\left[4(3x+1)^{\frac{1}{2}}\right]_0^{5}$ $=4\times\sqrt{16}-4\times\sqrt{1}$ $=12$ Therefore area A is half the total area, so area A = area B.	Correct integration.	Correct solution with correct integration.	

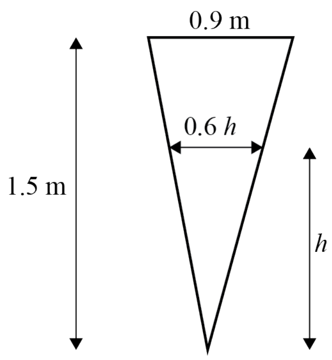
(e)	$\frac{\mathrm{d}N}{\mathrm{d}t} = kN$ $\int \frac{1}{N}\,\mathrm{d}N = \int k\,\mathrm{d}t$ $\ln N = kt + c$ $\ln N_1 = kt_1 + c$ $c = \ln N_1 - kt_1$ $\ln N_2 = k2t_1 + c$ $c = \ln N_2 - 2kt_1$ $\therefore \ln N_2 - 2kt_1 = \ln N_1 - kt_1$ $\ln N_2 - \ln N_1 = 2kt_1 - kt_1$ $\ln\left(\frac{N_2}{N_1}\right) = kt_1$ $k = \frac{1}{t_1}\ln\left(\frac{N_2}{N_1}\right)$	Separate variables and correct integration .	Correct equation involving N_1 and N_2.	Correct solution, presented in a coherent manner.

NØ	**N1**	**N2**	**A3**	**A4**	**M5**	**M6**	**E7**	**E8**
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q2	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$x + 0.5e^{4x} + c$	Correct solution.		
(b)	8.2	Correct value.		
(c)	$\int_3^k \frac{8}{2x-5}dx = \left[4\ln\lvert 2x-5\rvert\right]_3^k$ $= 4\ln\lvert 2k-5\rvert - 4\ln\lvert 1\rvert$ $= 4\ln\lvert 2k-5\rvert$ $\therefore 4\ln(2k-5) = 10$ $\ln(2k-5) = 2.5$ $k = \frac{e^{2.5}+5}{2} = 8.59$	Correct integration.	Correct solution with correct integration.	
(d)	$\int_0^\pi \cos^2 x\,dx = \frac{1}{2}\int_0^\pi (\cos 2x + 1)dx$ $= \frac{1}{2}\left[\frac{\sin 2x}{2} + x\right]_0^\pi$ $= \frac{1}{2}\left[\left(\frac{\sin 2\pi}{2} + \pi\right) - \left(\frac{\sin 0}{2} - 0\right)\right]$ $= \frac{\pi}{2}$	Correct integration.	Correct solution with correct integration.	
(e)	$(e^x)^2 = 20 - (e^x)^2$ $e^{2x} = 20 - e^{2x}$ $2e^{2x} = 20$ $e^{2x} = 10$ $x = \frac{\ln 10}{2} (= 1.151)$ Area = $\int_0^{\frac{\ln 10}{2}} (20 - e^{2x} - e^{2x})dx$ $= \int_0^{\frac{\ln 10}{2}} (20 - 2e^{2x})dx$ $= \left[20x - e^{2x}\right]_0^{\frac{\ln 10}{2}}$ $= \left(\frac{20 \times \ln 10}{2} - 10\right) - (0 - 1)$ $= 10\ln 10 - 9$ $(= 14.03)$	$20x - \frac{e^{2x}}{2}$	Correct integration. If done in 2 parts must put it together.	Correct solution with correct integration.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Q3	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$3(2x-1)^4+c$ OR $48x^4 - 96x^3 + 72x^2 - 24x + c$	Correct solution.		
(b)	$\frac{dy}{dx}=4\sec^2 2x$ $y=2\tan 2x+c$ $x=\frac{\pi}{8}\quad y=5$ $5=2\tan\left(\frac{\pi}{4}\right)+c$ $5=2+c$ $c=3$ $y=2\tan 2x+3$	Correct solution with correct integration. Possible N2: $y = 2\tan 2x$		
(c)	$\int_1^4 x+1+\frac{x}{x+1}dx=\int_1^4 x+1+1-\frac{1}{x+1}dx$ $=\int_1^4 x+2-\frac{1}{x+1}dx$ $=\left[\frac{x^2}{2}+2x-\ln(x+1)\right]_1^4$ $=(16-\ln 5)-(2.5-\ln 2)$ $=13.5-\ln 5+\ln 2$ $=12.58$ Using substitution: $\left[\frac{u^2}{2}+u-\ln u\right]_2^5=\left[\frac{(x+1)^2}{2}+x+1-\ln(x+1)\right]_1^4$ $=(17.5-\ln 5)-(4-\ln 2)$	Correct integration.	Correct solution with correct integration.	
(d)	$y=\int\left(\frac{4x}{4x^2-3}+\sqrt{x}\right)dx$ $=\int\left(\frac{1}{2}\left(\frac{8x}{4x^2-3}\right)+x^{\frac{1}{2}}\right)dx$ $=\frac{1}{2}\ln\left(4x^2-3\right)+\frac{2}{3}x^{\frac{3}{2}}+c$ $y(1)=2$ $\Rightarrow 2=\frac{1}{2}\ln(1)+\frac{2}{3}+c$ $c=\frac{4}{3}$ $y(4)=\frac{1}{2}\ln(61)+\frac{2}{3}\times 4^{\frac{3}{2}}+\frac{4}{3}$ $y=8.722$	Correct integration. Possible N2: $\frac{1}{2}in\left(4x^2-3\right)$	Correct solution with correct integration.	

<table>
<tr><td>(e)</td><td>Cross-section

$E = 9800\int_{H-d}^{H}(H-h)A(h)\,\mathrm{d}h$
$= 9800\int_{0.5}^{1.5}(1.5-h)0.36h^2\,\mathrm{d}h$
$= 9800\int_{0.5}^{1.5}(0.54h^2-0.36h^3)\,\mathrm{d}h$
$= 9800\left[0.18h^3-0.09h^4\right]_{0.5}^{1.5}$
$= 9800\left[(0.6075-0.4556)-(0.0225-0.0056)\right]$
$= 9800\left[0.1519-0.0169\right]$
$= 9800\times 0.135$
$= 1323\text{ J}$</td><td></td><td>Correct integration.</td><td>Correct solution with correct integration.</td></tr>
</table>

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE answer demonstrating limited knowledge of integration techniques.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 12	13 – 18	19 – 24