Assessment Schedule - 2020
Mathematics and Statistics: Apply calculus methods in solving problems (91262)

Evidence

| $\begin{gathered} \text { Q } \\ \text { ONE } \end{gathered}$ | Evidence | Achievement (u) | Achievement with Merit (r) | Achievement with Excellence (t) |
| :---: | :---: | :---: | :---: | :---: |
| (a) | $\begin{aligned} & f(x)=x^{3}-2 x^{2}+5 \\ & \Rightarrow f^{\prime}(x)=3 x^{2}-4 x \\ & f^{\prime}(4)=32 \end{aligned}$ | Correct solution. |  |  |
| (b) | $\begin{aligned} & h(x)=0.5 x^{2}+3 x-1 \\ & \Rightarrow h^{\prime}(x)=x+3 \\ & x+3=5 \Rightarrow x=2 \end{aligned}$ | Correct solution. |  |  |
| (c) | $\begin{aligned} & g^{\prime}(x)=2 x+5 \\ & g^{\prime}(2)=9 \\ & g(x)-14=9(x-2) \\ & g(x)=9 x-4 \end{aligned}$ | Finds the gradient. | Correct equation. |  |
| (d) | $\begin{aligned} & f^{\prime}(x)=p x-4 \\ & f(x)=\frac{p x^{2}}{2}-4 x+\mathrm{c} \\ & f(4)=8 p-16+\mathrm{c}=12 \Rightarrow \mathrm{c}=28-8 p \\ & f(-6)=18 p+24+\mathrm{c}=2 \Rightarrow \mathrm{c}=-22-18 p \\ & 28-8 p=-22-18 p \Rightarrow p=-5, \mathrm{c}=68 \\ & \text { Hence, } f(x)=\frac{-5 x^{2}}{2}-4 x+68 \end{aligned}$ | Integrates $f^{\prime}(x)$ correctly. | Correct equation for $f(x)$ |  |
| (e) | $\begin{aligned} & a(t)=0.5 \\ & v(t)=0.5 t+3 \\ & s(\mathrm{t})=0.25 t^{2}+3 t+80 \\ & s(t+1)-\mathrm{s}(t)=0.5 t+3.25=11.75 \mathrm{~km} \\ & t=17 \end{aligned}$ <br> So in the $18^{\text {th }}$ hour, the boat travels 11.75 km . | Finds correct velocity equation. | Finds correct distance equation. | T1: Forms equation for distance travelled in 1 hour but minor error made in working leading to a solution, such as an algebraic or numerical slip. <br> T2: Correct solution. |


| $\mathbf{N 0}$ | $\mathbf{N 1}$ | $\mathbf{N 2}$ | $\mathbf{A 3}$ | $\mathbf{A 4}$ | M5 | M6 | E7 | E8 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| No response; <br> no relevant <br> evidence. | A valid <br> attempt at one <br> question. | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | T1 | T2 |

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| $\begin{gathered} \mathrm{Q} \\ \text { TWO } \end{gathered}$ | Evidence | Achievement (u) | Achievement with Merit (r) | Achievement with Excellence (t) |
| :---: | :---: | :---: | :---: | :---: |
| (a) | Positive parabola with $x$-intercepts in the $3^{\text {rd }}$ square to the left of the $y$-axis and in the $7^{\text {th }}$ square to the right (not on a grid line). | Correct graph. |  |  |
| (b) | $\begin{aligned} & y^{\prime}(x)=6 x^{2}-84 x+240=0 \text { at turning points } \\ & 6\left(x^{2}-14 x+40\right)=0 \\ & (x-4)(x-10)=0 \\ & x=4 \text { or } 10 \end{aligned}$ <br> Correct use of any test to show that the maximum occurs at $x=4$, such as showing clearly that: <br> - $y(4)>y(10)$ <br> - $y^{\prime}(3.9)>0$ while $y^{\prime}(4.1)<0$ <br> - $y^{\prime \prime}(4)<0$ | $y^{\prime}(x)$ correct and set to 0 . | Clear reasoning to show that maximum occurs when $x=4$. |  |
| (c) | $\begin{aligned} & v(t)=3 t^{2}-5 t \\ & a(t)=6 t-5 \\ & a(2)=7 \end{aligned}$ | Correct answer. |  |  |
| (d) | Negative cubic with a minimum and $x$-intercept at $(\mathrm{c}, 0)$ and a maximum 10 squares to the right of the $x$-axis. Accept cubics that do not have point symmetry. | Negative cubic. | TPs correct. |  |
| (e) | $\begin{aligned} & f(x)=a x^{3}+b x^{2}+c x+d \\ & f^{\prime}(x)=3 a x^{2}+2 b x+c \\ & 27 a+6 b+c=0 \\ & 75 a-10 b+c=0 \end{aligned}$ <br> Eliminating $b \Rightarrow c=-45 a$ <br> OR <br> To have turning points at $x=3$ and $x=-5$, $\begin{aligned} & f^{\prime}(x)=k(x-3)(x+5) \\ & f^{\prime}(x)=k x^{2}+2 k x-15 k \\ & \therefore f(x)=\frac{k}{3} x^{3}+k x^{2}-15 k x+d \end{aligned}$ <br> So $a=\frac{k}{3} \Rightarrow k=3 a$ <br> and $c=-15 k \Rightarrow c=-45 a$ | Correct expression for $f^{\prime}(x)$ involving $a, b$ and $c$. | Forms the two equations utilising the TP information. | T1: Correct expression obtained but with incorrect working or incorrect mathematical statements. <br> T2: Correct expression obtained with correct working and mathematical statements. |


| N0 | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; no relevant evidence. | A valid attempt at one question | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | T1 | T2 |


| Q <br> THREE | Evidence | Achievement (u) | Achievement with Merit (r) | Achievement with Excellence (t) |
| :---: | :---: | :---: | :---: | :---: |
| (a) | $\begin{aligned} & f(x)=x^{3}-x^{2}-4 x+\mathrm{c}, \quad \mathrm{c}=4 \\ & f(x)=x^{3}-x^{2}-4 x+4 \end{aligned}$ | Correct answer. |  |  |
| (b) | $\begin{aligned} & \mathrm{A}=\pi r^{2}=\pi(0.7 t)^{2}=0.49 \pi t^{2} \\ & \frac{d A}{d t}=0.98 \pi t \\ & \begin{aligned} & \frac{d A}{d t}=0.98 \pi \times 20 \\ & \quad=19.6 \pi=61.58 \mathrm{~m}^{2} \mathrm{~s}^{-1} \\ & \text { (units not required) } \end{aligned} \end{aligned}$ | Expression for $\frac{d A}{d t}$. | Correct answer. |  |
| (c) | $\begin{aligned} & h^{\prime}(x)=\frac{-2}{3} x+2 \\ & h(x)=\frac{-1}{3} x^{2}+2 x+c \end{aligned}$ <br> Since $h(0)=4, h(x)=\frac{-1}{3} x^{2}+2 x+4$ | Anti-differentiation. | Correct answer. |  |
| (d) | $\begin{aligned} & f^{\prime}(x)=3 k x^{2}+9 \\ & f^{\prime}(2)=3 k(2)^{2}+9=15 \\ & 12 k+9=15 \\ & k=0.5 \end{aligned}$ | Correct answer. |  |  |
| (e) | Let length of the base be $x$. $\begin{aligned} A & =\frac{1}{2} x\left(m x-x^{2}\right) \\ & =\frac{1}{2} m x^{2}-\frac{1}{2} x^{3} \\ A^{\prime} & =m x-\frac{3}{2} x^{2}=x\left(m-\frac{3}{2} x\right)=0 \end{aligned}$ <br> Since $x \neq 0, \frac{3}{2} x=m \Rightarrow x=\frac{2 m}{3}$ <br> $\operatorname{Max~A:~} \quad A=\frac{m}{3}\left(\frac{2 m^{2}}{3}-\frac{4 m^{2}}{9}\right)=\frac{2 m^{3}}{27}$ | Correct differentiation of an expression for the area of OAB. | Finds $m$ or $x$. | T1: Minor error but otherwise correct. <br> T2: Correct expression obtained. |


| N0 | N1 | N2 | A3 | A4 | M5 | M6 | E7 | E8 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| No response; no relevant evidence. | A valid attempt at one question | 1 of u | 2 of u | 3 of u | 1 of r | 2 of r | T1 | T2 |

Cut Scores

| Not Achieved | Achievement | Achievement with Merit | Achievement <br> with Excellence |
| :---: | :---: | :---: | :---: |
| $0-8$ | $9-13$ | $14-18$ | $19-24$ |

