Assessment Schedule – 2021

Mathematics and Statistics: Apply probability methods in solving problems (91267)

Evidence

<table>
<tr><th>Q
ONE</th><th>Expected coverage</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr><td>(a)(i)</td><td>P(drink coffee) = $\frac{122}{300}$ = 0.4067</td><td>Probability correct.</td><td></td><td></td></tr>
<tr><td>(ii)</td><td>P(coffee if sleep issues) = $\frac{68}{143}$ = 0.4755</td><td>Probability correct.</td><td></td><td></td></tr>
<tr><td>(iii)</td><td>Jack is wrong since people who drink coffee are more likely to have sleep issues:
P(sleep issue if drink coffee) = $\frac{68}{122}$ = 0.5574
which is higher than
P(sleep issue if don't drink coffee) = $\frac{75}{178}$
= 0.4213
Jack is looking at the total numbers (75 having sleep issues with only 68 without sleep issues) but he needs to consider the proportions or risks out of the total in each category (not required).
(RR $\frac{0.5574}{0.4213}$ = 1.323 but this is not required)</td><td>One correct conditional probability.</td><td>2 correct conditional probabilities compared.
AND
Valid (brief) discussion of why Jack's statement was incorrect by comparing probabilities.</td><td></td></tr>
<tr><td>b (i)

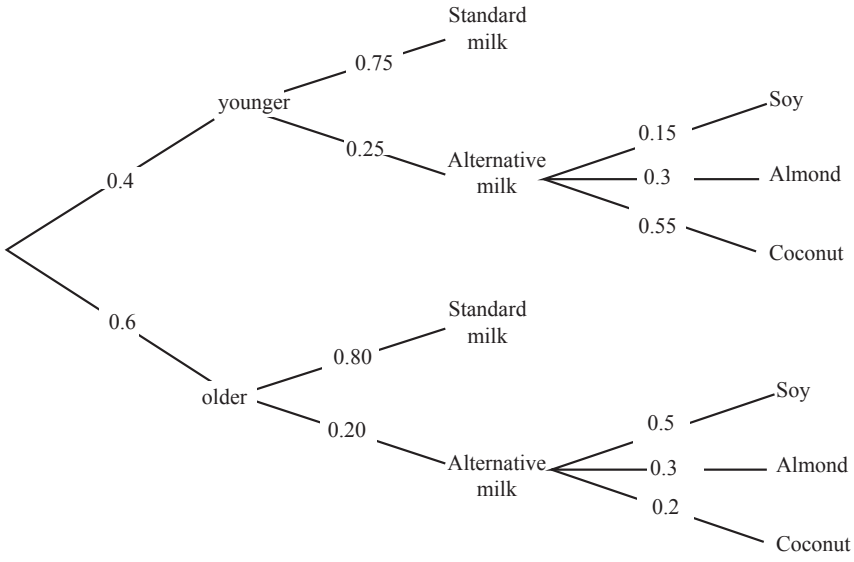
(ii)</td><td><table><tr><th></th><th>Have sleep issues</th><th>No sleep issues</th><th>Total</th></tr><tr><td>Drink coffee</td><td>68</td><td>54</td><td>122</td></tr><tr><td>Drink energy drinks (but not coffee)</td><td>29</td><td>7</td><td>36</td></tr><tr><td>Don't drink either coffee or energy drinks</td><td>46</td><td>103 – 7 = 96</td><td>178 – 36 = 142</td></tr><tr><td>Total</td><td>143</td><td>157</td><td>300</td></tr></table>P(sleep issues if neither caffeine drink) = $\frac{46}{142}$
= 0.3239
0.3239 × 850 = 275.35 so 275 students or 276 students 0r '275 or 276' students.</td><td>Correct Probability
OR
Error in table but consistent final answer (expected number of students) gets 'u'

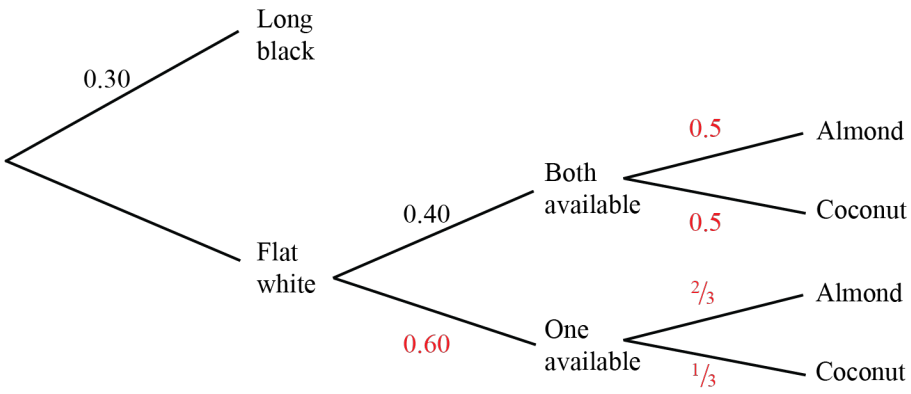
<u>Note:</u> This question (parts i and ii together) is for a single grade.</td><td>Table completed.
AND
Expected value found in (ii).
Must be whole number</td><td></td></tr>
</table>

(iii)	P(sleep issues if consume drinks containing caffeine) $= \frac{(68+29)}{(122+36)} = \frac{97}{158} = 0.6139$ P(sleep issues if don't consume drinks containing caffeine) $= \frac{46}{142} = 0.3239$ $\frac{0.6139}{0.3239} = 1.895$ So it is 1.89 times more likely (or 89% more likely) for students who consume drinks containing caffeine to have sleep issues than students who don't consume drinks containing caffeine. This is reasonably close to 2 so it is a valid claim (since the article says **nearly** twice as likely). **OR** This is less than 2 so the claim of twice as likely is not valid **(Award T1 for getting this far)** Comments about validity of survey (**for T2**) However, it may not be valid because: • this was an online survey of only 300 students at one school, so while it is a reasonable sample size, it may be biased / not representative of all NZ students • coffee and energy drinks aren't the only source of caffeine • any other valid reason.	Correct probability of sleep issues if consume caffeinated drinks.	Relative risk found (or sensible multiplicative comparison). **OR** Relative risk interpreted in context but looking only at coffee **or** energy drinks (**one** row of the table) but not both rows combined.	T1: relative risk correct and interpreted in context. T2: Relative risk correct and interpreted in context AND discussion of validity of claim with at least one reasonable point made.

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	A valid attempt at one question.	1 of u	2 of u	3 of u	1 of r	2 of r	T1	T2

Q TWO	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)(i)	P(younger and standard milk) = 0.4 × 0.75 = 0.3	Correct probability. Tree not required.		
(ii)	P(soy) = P(younger and alt and soy) = 0.4 × 0.25 × 0.15 + P(older and alt and soy) 0.6 × 0.2 × 0.5 = 0.015 + 0.06 = 0.075	One correct probability.	Correct probability added.	
(iii)	P(alternative milk) = 0.4 × 0.25 + 0.6 × 0.2 = 0.1 + 0.12 = 0.22 P(soy if alternative) = $\frac{0.075}{0.22} = 0.3409 = 34.1\%$	P(alternative milk) found (denominator).	Correct proportion / probability – does **not** have to be a percentage.	
(iv)	0.4 × 0.25 × x + 0.6 × 0.2 × x = 0.066 0.1x + 0.12x = 0.066 0.22x = 0.066 x = 0.3 P(customer orders cow's milk) = 0.4 × 0.75 + 0.6 × 0.8 = 0.78 P(customer orders coconut milk) = 0.4 × 0.25 × 0.55 + 0.6 × 0.2 × 0.2 = 0.079 RR = $\frac{0.78}{0.079}$ = 9.873 so customers are 9.9 (9.8) times as (more) likely to order cow's milk than coconut milk. Accept any combination of RR and as or more. Any other valid method.	Correct P(cow's milk) – 0.78 OR CAO for x with evidence of trial and error. OR Tree set up correctly with x on both almond branches.	Correct value of x found.	T1: Correct value of x found and correct probability for P(coconut milk). T2: Relative risk calculated and interpreted for P(coconut) compared to P(cow's) milk.



(b)	$\frac{1}{3}$ and $\frac{2}{3}$ deduced. P(almond-milk flat white) $= 0.7 \times 0.4 \times 0.5 + 0.7 \times 0.6 \times \frac{2}{3} = 0.14 + 0.28$ $= 0.42$ P(coconut-milk flat white) $= 0.7 \times 0.4 \times 0.5 + 0.7 \times 0.6 \times \frac{1}{3} = 0.14 + 0.14$ $= 0.28$ P(long black) = 0.3 Kathy is more likely to have an almond-milk flat white than coconut or long black.  	Either probability of almond or coconut-milk flat white found.	P(flat white) for almond and coconut milk found correctly and correct conclusion. ***Any*** justification sufficient – eg circling the Almond-milk Flat White probability.	

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	A valid attempt at one question.	1 of u	2 of u	3 of u	1 of r	2 of r	T1	T2

Q THREE	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
(a)	$P(X < 5) = P(Z < -1.333) = 0.0912$ (0.0913 if tables used)	Correct proportion.		
(b)	$P(8 < X < 10) = P(0.667 < Z < 2) = 0.2297$ (0.2295 from table) $0.2297 \times 150 = 34.455$ so 34 customers, or 35 customers, or '34 or 35 customers'.	Correct probability. **OR** CAO	Correct number of customers. Must be whole number.	
(c)	$P(X < 5) = 0.3$ $P(Z < z) = 0.3 \quad z = -0.5244$ $-0.5255 = \frac{(5-7)}{\sigma}$ $\sigma = 3.814$ minutes This means they have a higher standard deviation than the café as a whole, so they are less consistent (more variable). Therefore, while they have a higher proportion of customers who wait under 5 mins, they would also have a higher proportion who wait a long time, so I don't think they should be rewarded.	CAO OR Correct z-value found (± 0.5244).	Correct standard deviation found.	Correct standard deviation AND a discussion of what this means about the staff member's consistency.

<table>
<tr>
<td>(d)(i)</td>
<td>Inverse normal P(LQ < X < UQ) = 0.50
central 50% P(–0.6745 < Z < 0.6745) = 0.5
<table>
<tr><th>Statistic</th><th>Data from Figure 1</th><th>Normal model</th></tr>
<tr><td>Median</td><td>6</td><td>7</td></tr>
<tr><td>Lower quartile</td><td>5</td><td>5.99</td></tr>
<tr><td>Upper quartile</td><td>8.5</td><td>8.01</td></tr>
<tr><td>Interquartile range</td><td>3.5</td><td>2.02</td></tr>
</table>
</td>
<td>Evidence of $|z| = 0.6745$
OR
labelled sketch indicating middle 50%.
OR
CAO.</td>
<td>Quartiles found for normal model.</td>
<td></td>
</tr>
<tr>
<td>(ii)</td>
<td>Centre: Compares means or medians
Café data median (6) is lower than normal model (7) so the model does not fit the data (merit)
A different normal model with a mean of 6 might fit better (extra for excellence)
OR
A normal distribution has mean=median. This data is skewed to the right, so the mean≠median (merit)
Therefore this data cannot be normally distributed (extra for excellence)
Spread: Compares IQR, Range, or σ
The IQR is much higher than the model, suggesting the data is more spread out than model (merit).
The expected range would be ±4.5 (3 × 1.5) from the mean so from 2.5 to 11.5, but the data goes from 1.5 to 15 (indicating standard deviation would be about 2.25 rather than 1.5) (extra justification for excellence).
Shape: The data is clearly not symmetrical and unimodal, which would be expected for a normal distribution (merit).
It is skewed to the right (higher chance of really long time to make coffee than model suggests) and has a peak at 5 and 7 minutes, indicating bimodal (possibly due to multiple coffee orders), as well as an unusual cluster at 15 minutes (extra justification for excellence).

Normal distribution model would also go lower than 0, which is impossible in this context. <u>Note:</u> This comment is limited to r grade only.</td>
<td></td>
<td>Two valid comments comparing centre, spread or shape of the data to the model.</td>
<td>At least two different valid comments <u>comparing</u> centre, spread, or shape of the data to the expected model, with context or evidence explaining why the normal model is or isn’t appropriate to this context.
<u>Note:</u> Final statement on appropriateness of model not required.</td>
</tr>
</table>

N0	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	A valid attempt at one question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 14	15 – 19	20 – 24

Notes:

- Allow *any* correct truncation or rounding throughout.
- In all Normal Distribution calculations allow z-values to 2 or more decimal places.
- For r or t in Q3 (b) to (d) some working (calculation or labelled / shaded diagram) is required.