Assessment Schedule – 2021

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$\frac{x^2}{6}+3\ln\lvert x\rvert+c$	Correct integral + c not required.		
(b)(i)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{8}{x^3}$ $y=\int 8x^{-3}\mathrm{d}x$ $y=-4x^{-2}+c$ $3=-4+c$ $c=7$ $y=-4x^{-2}+7$	Correct solution with correct integral.		
(ii)	$y=-4x^{-2}+7$ Area = $\int_1^2\left(-4x^{-2}+7\right)\mathrm{d}x$ $=\left[4x^{-1}+7x\right]_1^2$ $=\left(\frac{4}{2}+14\right)-\left(\frac{4}{1}+7\right)$ $=5$	Correct integral. (Limits not needed) CC: Integration of $y=-4x^{-2}+c$ From (b)(i).	Correct solution with correct integral. CC: Accept if correct solution for curve with any + c $y=-4x^{-2}+c$ From (b)(i).	
(c)	$a(t)=2-\sin 2t$ $v(t)=\int\left(2-\sin 2t\right)\mathrm{d}t$ $=2t+\frac{\cos 2t}{2}+c$ $1=0+\frac{\cos 0}{2}+c$ $c=0.5$ $v(t)=2t+\frac{\cos 2t}{2}+0.5$ $s(t)=\int\left(2t+\frac{\cos 2t}{2}+0.5\right)$ $s(t)=t^2+\frac{\sin 2t}{4}+0.5t+k$ $t=0, s=3\Rightarrow k=3$ $s(5)=5^2+\frac{\sin 10}{4}+0.5\times 5+3$ $=30.5+\frac{1}{4}\sin 10$ $=30.36$	Correct expression for $v(t)$.	Correct solution with correct integrals.	

(d)	$\frac{dV}{dt} = k\sqrt{V}$ $\int \frac{1}{\sqrt{V}} dv = \int k\, dt$ $\int V^{-0.5} dv = \int k\, dt$ $2\sqrt{V} = kt + c$ $T = 6,\ V = 400,\ 40 = 6k + c$ $T = 10,\ V = 256,\ 32 = 10k + c$ $k = -2 \quad c = 52$ $\therefore 2\sqrt{V} = -2t + 52$ $t = 0$ $2\sqrt{V} = 52$ $\sqrt{V} = 26$ $V = 676$ (litres)	Correct general solution to DE. + c not required.	Correct general solution to DE plus correct values of c and k.	Correct solution (units not required)

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\frac{e^{4x}}{4}+\frac{8}{3}x^{\frac{3}{2}}+c$	Correct integral + c not required.		
(b)	14	Correct solution.		
(c)	$\int_0^{\frac{\pi}{8}}\sin 6x\cdot\sin 2x\,dx=\frac{1}{2}\int_0^{\frac{\pi}{8}}(\cos 4x-\cos 8x)dx$ $=\frac{1}{2}\left[\frac{\sin 4x}{4}-\frac{\sin 8x}{8}\right]_0^{\frac{\pi}{8}}$ $=\frac{1}{2}\left[\left(\frac{\sin\frac{\pi}{2}}{4}-\frac{\sin\pi}{8}\right)-0\right]$ $=\frac{1}{8}$	Correct integration. (Limits not needed)	Correct solution with correct integration.	
(d)	$\int_2^k \frac{6}{3x-4}dx=\left[2\ln(3x-4)\right]_2^k$ $=2\ln(3k-4)-2\ln(2)=4$ $2\ln(3k-4)=4+2\ln(2)$ $\ln(3k-4)=2+\ln(2)$ $3k-4=e^{2+\ln 2}$ $3k-4=2e^2$ $k=\frac{2e^2+4}{3}$ $k=6.26$	Correct integration. (Limits not needed)	Correct solution with correct integration. Accept $k=\frac{2e^{2+\ln 2}+4}{3}$.	
(e)	$\frac{dy}{dx}=\frac{2}{ye^{0.5x}}$ $\int y\,dy=\int 2e^{-0.5x}dx$ $\frac{y^2}{2}=-4e^{-0.5x}+c$ $x=0$ and $y=1$ $\frac{1}{2}=-4+c$ $c=4.5$ $\frac{y^2}{2}=-4e^{-0.5x}+4.5$ At $x=3$: $\frac{y^2}{2}=-4e^{-1.5}+4.5$ $y=2.686$ Distance $=5.372$	Correct integration with correct rearrangement of both sides. + c not required.	Correct solution of DE.	Correct solution with correct integrations shown. E7 $y=\pm 2.686$

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\int\left(x+\sqrt{x}\right)^2 \mathrm{d}x$ $=\int\left(x^2+2x\sqrt{x}+x\right)\mathrm{d}x$ $=\int\left(x^2+2x^{\frac{3}{2}}+x\right)\mathrm{d}x$ $=\frac{x^3}{3}+\frac{4}{5}x^{\frac{5}{2}}+\frac{x^2}{2}+c$	Correct integral + c not required.		
(b)	2.6	Correct solution.		
(c)	$\frac{\mathrm{d}y}{\mathrm{d}x}=\frac{\sec^2 2x}{y}$ $\int y\,\mathrm{d}y=\int\sec^2 2x\,\mathrm{d}x$ $\frac{y^2}{2}=\frac{\tan 2x}{2}+c$ $x=\frac{3\pi}{4}, y=2$ $\frac{2^2}{2}=\frac{\tan\frac{3\pi}{4}}{2}+c$ $2=\frac{-1}{2}+c$ $c=2.5$ $\frac{y^2}{2}=\frac{\tan 2x}{2}+2.5$ $x=\pi$ $\frac{y^2}{2}=\frac{\tan 2\pi}{2}+2.5$ $\frac{y^2}{2}=2.5$ $y=\pm\sqrt{5}=\pm 2.236$	Correct integration after correct separation of both sides. (+ c not required)	Correct solution with correct integration after correct separation of both sides (± not required)	
(d)	$y=\frac{3x-2}{x+2}$ $y=1\Rightarrow x=2 \quad y=2\Rightarrow x=6$ $\frac{3x-2}{x+2}=3-\frac{8}{x+2}$ $\int_2^6\left(3-\frac{8}{x+2}\right)\mathrm{d}x$ $=\left[3x-8\ln(x+2)\right]_2^6$ $=(18-8\ln 8)-(6-8\ln 4)$ $=12+8\ln 0.5$ (or $12-8\ln 2$) $=6.455$	Correct integration. (Limits not needed.)	Correct solution with correct integration.	

(e)	$\left(ke^x\right)^2 = k$ $k^2e^{2x} = k$ $e^{2x} = \frac{1}{k}$ $x = \frac{\ln\frac{1}{k}}{2}$ $\text{Area} = k^2\int_{\frac{\ln\frac{1}{k}}{2}}^{0} e^{2x}\,dx - \int_{\frac{\ln\frac{1}{k}}{2}}^{0} k\,dx$ $= \frac{k^2}{2}\left[e^{2x}\right]_{\frac{\ln\frac{1}{k}}{2}}^{0} - \left[kx\right]_{\frac{\ln\frac{1}{k}}{2}}^{0}$ $= \frac{k^2}{2}\left(1 - e^{2\times\frac{\ln\frac{1}{k}}{2}}\right) - \left(0 - k\cdot\frac{\ln\frac{1}{k}}{2}\right)$ $= \frac{k^2}{2}\left(1 - \frac{1}{k}\right) + \frac{k}{2}\cdot\ln\frac{1}{k}$ $= \frac{k^2}{2} - \frac{k}{2} + \frac{k}{2}\cdot\ln\frac{1}{k}$ $= \frac{k}{2}\left(k - 1 + \ln\frac{1}{k}\right)$ OR $\text{Area} = \int_{\frac{\ln\frac{1}{k}}{2}}^{0}\left(k^2e^{2x} - k\right)dx$ $= \left[\frac{k^2}{2}e^{2x} - kx\right]_{\frac{\ln\frac{1}{k}}{2}}^{0}$ $= \left(\frac{k^2}{2} - 0\right) - \left(\left[\frac{k^2}{2}e^{\ln\frac{1}{k}} - \frac{k^2}{2}\ln\frac{1}{k}\right]\right)$ $= \frac{k^2}{2} - \frac{k}{2} + \frac{k}{2}\ln\frac{1}{k}$ $= \frac{k}{2}\left(k - 1 - \ln\frac{1}{k}\right)$	Correct integration. (Limits not needed.)	Correct integration with correct limits substituted in.	Correct solution with steps clearly shown.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 13	14 – 19	20 – 24