Assessment Schedule – 2022

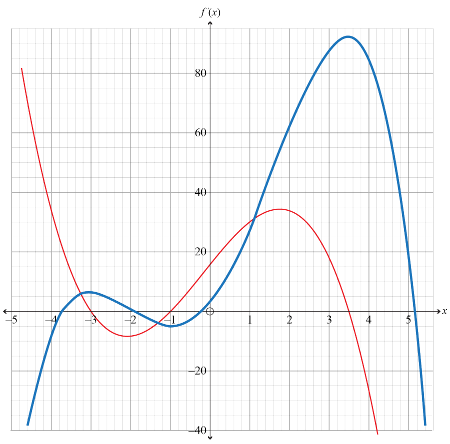
Mathematics and Statistics: Apply calculus methods in solving problems (91262)

Evidence

Q	Evidence	Achievement	Merit	Excellence
ONE (a)	$f'(x) = 8x^3 + 12x^2 - 40x$ $f'(3) = 8(3)^3 + 12(3)^2 - 40(3)$ $f'(3) = 204$	Derivative found and gradient evaluated.		
(b)	$f(x) = 4x - \frac{6x^2}{2} + \frac{2x^3}{3} + C$ At $(3,4)$ $4 = 4(3) - 3(3)^2 + \frac{2(3)^2}{3} + C$ $C = 1$ $f(x) = 4x - \frac{6x^2}{2} + \frac{2x^3}{3} + 1$	Equation of $f(x)$ found.		
(c)	$f'(x) = 2x^2 + 3x - 20$ is decreasing when $f'(x) < 0$ $2x^2 + 3x - 20 < 0$ $-4 < x < 2.5$ OR as $2.5 < x < -4$ or $x < -4$ AND $x < 2.5$	Derivative found and made < 0 or $= 0$.	Correct interval found.	
(d)	$f(x) - px - qx^2$ $f'(x) = p - 2qx$ When $x = 2, f'(x) = -6$ $-6 = p - 2q(2)$ At $(2,-10)$ $-10 = p(2) - q(2)^2$ $-10 = 2p - 4q$ $q = 0.5$ $p = -4$	Derivative found. AND Substitution of $x = 2$ into $f'(x)$. AND $f'(2) = -6$.	Two linear equations found. OR Substitution used to eliminate either p or q.	Correct solutions for BOTH p and q found.

(e)	$A = 2rh - \pi r^2$ Limiting constraint $2\pi r + 2h = 80$ OR $h = 40 - \pi r$ $A = 2r(40 - \pi r) - \pi r^2$ $A = 80r - 3\pi r^2$ $A' = 80 - 6\pi r$ $A' = 0$ $0 = 80 - 6\pi r$ $r = \frac{40}{3\pi} = 4.244$ (3 d.p.) Max area $= 80(4.24) - 3\pi(4.24)^2$ $= 169.77\text{ cm}^2$ (2 d.p.) Justification $A'' = -6\pi$ Second derivative is always negative $\rightarrow$ max.	Sets up area equation in terms of a variable AND differentiates.	Makes A′= 0 AND solves for *r*.	Finds max area, with justification. Justification from: Gradient function or Second derivative function, gradient on each side of the point.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	One point made incompletely.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Q	Evidence	Achievement	Merit	Excellence
TWO (a)	 Blue line is $f(x)$ (Answer to this Q) Red line is the original gradient function $f'(x)$.	Negative Quartic (x^{-4}) assumed. Note: If axis not relabelled, then an inverted parabola is drawn f'(x) = 0 at $x = -2$ AND just to the left of $x = 2$.	Negative Quartic (x^4) shape with turning points lined up with x-intercepts of given gradient graph. 3 min/max total Local max: $x = -3$ Local min : $x = -1$ Local max : $x = 3.5$ A reasonable quartic shape. Note: If an inverted parabola is drawn the vertex is just to the left of $x = 0$ and of good shape → 'r'.	Local max at $x = 3.5$ with a higher y-value than the local max at $x = -3$, due to steeper slope of gradient function. AND a good continuous quartic shape.
(b)	$h'(t) - 22.5 - 9.8t$ At max $h'(x) = 0$ $0 = 22.5 - 9.8t$ $t = \frac{22.5}{9.8} = 2.296$ (3 d.p.) $h(2.3) = 22.5(2.3) - 4.9(2.3)^2 + 1 = 26.83m$ (2 d.p.)	Derivative found and equated to 0.	Height found.	
(c)(i) (ii) (iii)	$P'(t) = 200t - 8t^3$ $P'(6) = 200(6) - 8(6)^3$ $P'(6) = -528$ people / hour Means that <u>528 people</u> per hour were <u>leaving</u> the game <u>at that time</u>. $P' = 2kt - 8t^3$ $P'' = 2k - 24t^2$ $P'' = 2k - 24t^2 = 0$ $P'' = 0$ for max $2k = 24t^2$ $k = 12t^2$ if $t = 4$, $k = 192$	$\frac{dP}{dt}$ found. AND $t = 6$ substitution shown. P'' found.	 Correct interpretation. P'' set = 0.	 $P'' = 0$ for max. AND Solves to find $k = 192$ with correct statements.

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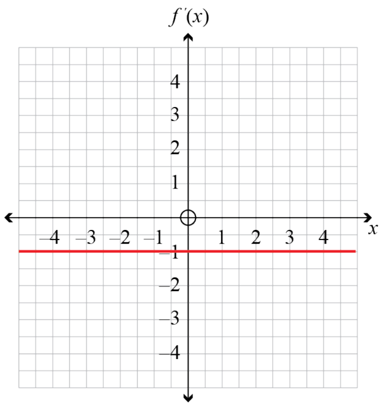
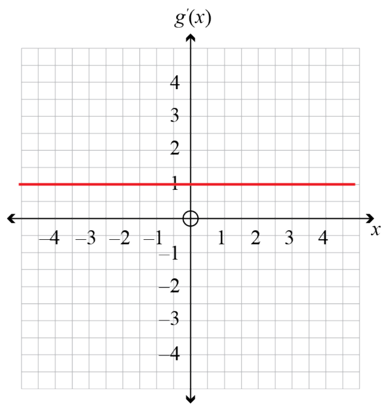
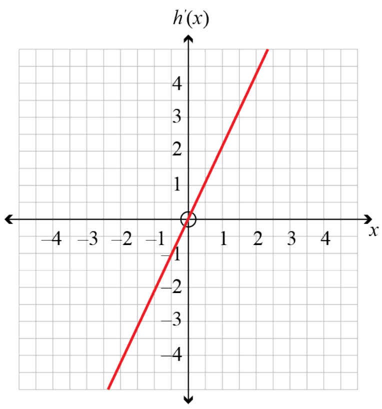
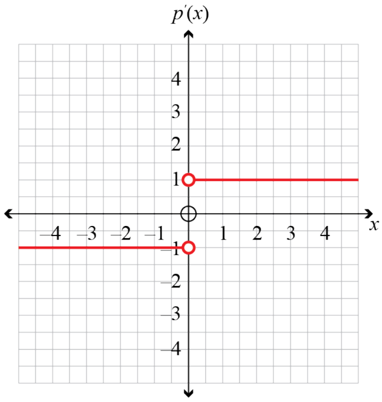
No response; no relevant evidence.	One point made incompletely.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Q	Evidence	Achievement	Merit	Excellence
THREE (a)	$y = 2x(x-3)$ $y = 2x^2 - 6x$ $y' = 4x - 6$ $y'(1) = 4(1) - 6$ $y'(1) = -2$ Equation of the tangent $(y-4) = -2(x-1)$ $y = -2x - 2$	Correct derivative found.	Equation of tangent found.	
(b)(i)	  	At least 2 gradient functions drawn correctly.		

(ii)		Gradient function sketched with no open circles on $x = 0$. Note: Joins vertical line at $x = 0$ on the range $y \in [-1,1] \rightarrow$ 'ns'.	Gradient function sketched correctly with open circles on $x = 0$.	
(iii)	The gradient function has a discontinuity, break, gap when $x = 0$. Instantaneous change from negative slope to positive slope, causing undefined gradient function at $x = 0$.	On the journey. e.g. $y = -x$, $y' = -1$ and $y = x$, $y' = 1$, so at (0.0) it is not fair because gradients are not the same.	Discontinuity / gap or break at $x = 0$.	Description of reasoning of why the function not being differentiable at $x = 0$. Description of the issues drawing 'a tangent' at a sharp point.
(c)	$SA = \sqrt{3}a^2 + 2\sqrt{3}b^2$ Length of edges: $6a + 12b = 180$ $a = \frac{180 - 12b}{6}$ $a = 30 - 2b$ $SA = \sqrt{3}a^2 + 2\sqrt{3}b^2$ $SA = \sqrt{3}(30 - 2b)^2 + 2\sqrt{3}b^2$ $SA = 6\sqrt{3}b^2 - 120\sqrt{3}b + 900\sqrt{3}$ $SA' = 0$ for max or min $0 = 12\sqrt{3}b - 120\sqrt{3}$ $b = 10$ $a = 30 - 2b$ $a = 10$ Justification: $SA'' = 12\sqrt{3} > 0$ Second derivative is always positive $\rightarrow$ minimum.	Equation for the SA is formed and substitution made to eliminate either a or b.	Derivative taken, and set $SA' = 0$.	Solve for both a and b. AND Justification from: Graph of function or gradient function, gradient on each side of the points, or second derivative test.

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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 7	8 – 13	14 – 18	19 – 24