Assessment Schedule – 2022

Mathematics and Statistics: Apply probability methods in solving problems (91267)

Evidence

Q	Evidence	Achievement	Merit	Excellence
ONE (a)	Normal distribution $\mu = 227$ $\sigma = 16$ $p(227 < X < 247) = p(0 < Z < 1.25)$ $= 0.3944$	Probability correct.		
(b)(i)	$p(X < 210) = p(Z < -1.0625) = 0.1440$ $p(\text{both}) = 0.144 \times 0.144 = 0.0207$	Probability of 0.1440	Probability of 0.0207 AND with some reason explaining that the probability will be higher	
(ii)	If people have water shortages, they probably use less water than average, **so $p(X < 210)$ will be higher** than whole of New Zealand, so expect answer to be higher.			
(c)	Suzanne's results are **compared** to a normal distribution. Centre #1: Median of this data (approx. 8) is about the same (8) as a normal model. Centre #2: Mean (9.16) of this data is higher than mean (8) of the normal distribution so the means are not equal. Centre #3: In a normal distribution mean / median / mode will be all the same (8) but in this data they are not all the same. Shape: Normal model is bell-shaped and / or symmetrical, Suzanne's data is skewed (to the right). Could include comments about comparing peaks / mode. Spread: Normal model s.d. is 2 suggesting range of about 2 – 14 (± 3 s.d.) while Suzanne's data has a larger range (of 20) suggesting a larger s.d.	ONE valid comparison of normal model to data.	TWO valid comparisons of model to data for at least two of centre, spread, and shape.	
(d)	Inverse normal $\mu = 8$ $\sigma = 2$ $p(X < x) = 0.15$ (left tail) $x = 5.93$, so showers under 6 minutes are 'acceptable'.	Finding $z = -1.036$ (allow $z = 1.036$) OR CAO	Inverse normal used to find value with working and / or diagram.	

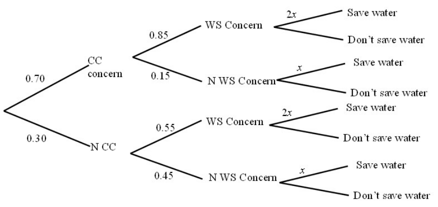
(e)(i) (ii)	$p(X < 2000) = 0.85$ $p(Z < z) = 0.85 \quad z = 1.036$ $1.036 = \frac{2000 - \mu}{450}$ #1 $\mu = 1533.62$ L per day Either mean and / or standard deviation must be higher or else it is clearly not a normal distribution shape. e.g. $p(X > 15\,000)$ in this model (with μ=1533, $\sigma = 450$) would be way less than 1.5% (has z-score of 29.9), suggesting that the shape of this data may be very skewed to the right (top 1.5% would be above 2510 litres which is considerably below 15 000).	Finding $z = 1.036$ OR CAO	Incorrect mean found with valid working, which must include #1. OR Valid discussion relating to e) (ii)	T1: mean found with valid working and / or diagram. T2: Mean found with valid working and / or diagram AND discussion of why normal distribution shape or these parameters would not be valid (justified with some numerical evidence).

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	One partial solution	1 of u	2 of u	3 of u	1 of r	2 of r	t1	t2

Q	Evidence	Achievement	Merit	Excellence
TWO (a)(i)	p(urban and unsafe for swimming) $= \frac{57}{810} = \frac{19}{270} = 0.0704$	Correct probability.		
(ii)	p(acceptable if native vegetation) $= \frac{146}{194} = \frac{73}{97} = 0.7526$ p(acceptable if exotic forest area) $= \frac{15}{26} = 0.5769$ Evidence that the probabilities have been compared (in words or numerically) to justify that a Native Vegetation river is more likely to be acceptable and safe for swimming as it has a higher probability (Could include Relative Risk = 1.3045 times as likely)	One correct conditional probability.	Both conditional probabilities correct AND conclusion based on comparison (Relative Risk not needed).	
(iii)	p(native river is unsafe) $= \frac{48}{194} \times 0.48 = 0.1188$ p(exotic river is unsafe) $= \frac{11}{26} \times 0.05 = 0.0212$ p(pasture river is unsafe) $= \frac{424}{528} \times 0.46 = 0.3694$ p(urban river is unsafe) $= \frac{57}{62} \times 0.01 = 0.00919$ p(unsafe) = 0.1188 + 0.0212 + 0.3694 + 0.00919 = 0.5186 = 51.86 %	At least one correct probability found. i.e. $\frac{48}{194} = 0.2472$ or $\frac{11}{26} = 0.4231$ or $\frac{424}{528} = 0.8030$ or $\frac{57}{62} = 0.9194$	At least one probability of river being unsafe for swimming found. i.e. 0.1188 or 0.0212 or 0.3694 or 0.00919	Correct probability of unsafe for swimming 0.5186 or 51.86 % AND a reasoned discussion on why the sample is / isn't representative.
(iv)	Not Confident # 1 because : Table 1 is based on one limited, possibly unrepresentative sample of rivers, since the profile of land areas does not match that for the whole country. (e.g. The sample says that Native Vegetation $\frac{194}{810} = 24\%$ which does not match the 48% in Native Vegetation of the actual proportion of NZ) Not Confident # 2 because: In addition, things may have changed in the two years since the survey was done. Therefore, it is probably unwise to think that the results are accurate two years later. I am Confident # 1 because: The sample size of 810 is large and the data has been collected by LAWA so this would support the confidence level.			

(b)(i)	p(river is safe) = $\frac{170}{278} = \frac{85}{139} = 0.6115$	Correct probability.		
(ii)	p(unsafe if NI) = $\frac{74}{179}$ = 0.4134 p(unsafe if SI) = $\frac{34}{99}$ = 0.3434 Joe found the relative risk $\frac{0.4134}{0.3434} = 1.204$ so this is only 20 % more likely for NI to have an unsafe river site than the SI. Therefore, Joe's reasoning is correct mathematically. Mia is looking at the number of unsafe rivers (74 is more than double 34), but this is not valid as the totals are different. She needed to look at proportions.	One correct conditional probability.	Relative risk found (or sensible multiplicative comparison).	T1 : Interpreting the relative risk in context AND explaining why Joe is correct or Mia wrong. T2 : Interpreting the relative risk in context AND explaining both Mia and Joe's reasoning

NØ	**N1**	**N2**	**A3**	**A4**	**M5**	**M6**	**E7**	**E8**
No response; no relevant evidence.	One partial solution	1 of u	2 of u	3 of u	1 of r	2 of r	t1	t2

Q	Evidence	Achievement	Merit	Excellence
THREE (a)(i)	p(concerned and usually save water) = $0.87 \times 0.8 = 0.696 = \frac{87}{125}$	Correct proportion.		
(ii)	p(save water if restrictions) $= 0.87 \times 0.18 + 0.13 \times 0.25$ $= 0.1566 + 0.0325$ $= 0.1891$ Expected number $= 0.1891 \times 2500$ $= 472.75$, so 473 (or 472) respondents.	Correct probability. OR CAO	Correct number of respondents. Must be whole number.	
(b)(i)	 p(Water Shortage Concern) $= 0.7 \times 0.85 + 0.3 \times 0.55 = 0.76$	Correct probability. (Evidence of the probability tree is not necessary.)		
(ii)	$0.7 \times 0.85 \times 2x + 0.7 \times 0.15 \times x + 0.3 \times 0.55 \times 2x + 0.3 \times 0.45 \times x = 0.5632$ $1.19x + 0.105x + 0.33x + 0.135x = 0.5632$ $1.76x = 0.5632$ $x = 0.32$ p(none) $= 0.3 \times 0.45 \times 0.68 = 0.0918$		Incorrect x-value found from correct process with minor misconception e.g. only one pair of branches considered.	T 1: Correct x-value found with valid working (allow minor error) T 2: Correct probability found.

(iii)	p(save water in secondary school survey) = 0.5632 p(save water in general survey) $= 0.87 \times 0.8 + 0.13 \times 0.5 = 0.761$. Relative Risk $= \frac{0.5632}{0.761} = 0.74$ Probability of student saving water is 0.74 times as likely – which is 26% less likely than the probability of general NZ population saving water. This is about 25% less likely, so the claim could be valid. OR **(alternative interpretation)** p(save water in general survey) $= 0.87 \times 0.8 + 0.87 \times 0.18 + 0.13 \times 0.5 + 0.13 \times 0.25$ $= 0.9501$ $= 0.87 \times 0.8 + 0.87 \times 0.18 + 0.13 \times 0.5 + 0.13 \times 0.25$ $= 0.9501$ Relative Risk $= \frac{0.5632}{0.9501} = 0.5928$ Probability of student saving water is 0.5928 times as likely – which is 40% less likely than the probability of general NZ population saving water. So the claim is not valid. The validity could be questioned because: #1 The second survey was only surveying students from two local schools, so would not represent all NZ students – for example students in cities are likely to have different opinions and actions than students in rural areas, therefore the claim may not be valid. # 2 There are also differences in how people who save water were categorised, making comparison of the surveys difficult. # 3 On-line survey maybe biased.	Probability of 0.761 found. OR (using the alternative interpretation) Probability of 0.9501 found.	Probabilities compared using Relative Risk (or using sensible multiplicative comparison). OR Correct probability of 0.761 calculated. AND validity of claim discussed.	Correct Relative Risk found AND Conclusion made regarding the claim. (or using sensible multiplicative comparison). OR Correct Relative Risk found AND with clear discussion of validity of claim.

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No response; no relevant evidence.	One partial solution	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 8	9 – 13	14 – 19	20 – 24