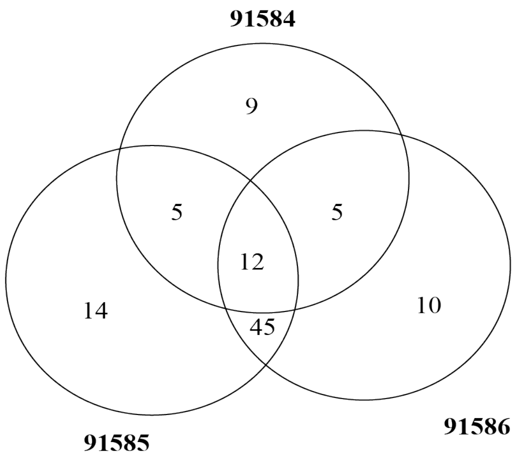
Assessment Schedule – 2022

Mathematics and Statistics (Statistics): Apply probability concepts in solving problems (91585)

Evidence Statement

<table>
<tr><th>Q</th><th>Expected Coverage</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr><td>ONE
(a)</td><td><table><tr><td></td><td>Technology</td><td>Not a Technology</td><td></td></tr><tr><td>Language</td><td>10</td><td>19</td><td>29</td></tr><tr><td>Not a Language</td><td>52</td><td>86</td><td>138</td></tr><tr><td></td><td>62</td><td>105</td><td>167</td></tr></table>$P(T' \cap L) = \frac{19}{167} = 0.114$</td><td>• Correct probability.</td><td></td><td></td></tr>
<tr><td>(b)</td><td>P(one student not studying a language if studying technology) $= \frac{52}{62}$.
So, P(all three students not studying a language if studying a technology)
$= \frac{52}{62} \times \frac{51}{61} \times \frac{50}{60} = 0.584$ (3 s. f.)</td><td>• Correct probability for first student.</td><td>• Correct conditional probability for all three students found.</td><td></td></tr>
<tr><td>(c)</td><td>Method 1: comparison of P(T∩L) and P(T) × P(L)
$P(T)) = \frac{62}{167}$ $P(L) = \frac{29}{167}$ $P(T \cap L) = \frac{10}{167} = 0.060$
$P(T) \times P(L) = \frac{62}{167} \times \frac{29}{167} = 0.064 \neq 0.060$
Different answers suggest non independence of events.
We have insufficient evidence to support the school's claim.
Studying one of these subjects affects the probability of studying the other subject.
Accept alternative arguments . e.g. Comparison of conditional probabilities.</td><td>• Correct probabilities calculated as part of a reasonable attempt to use an independence argument.</td><td colspan="2">• From the three aspects below award grade:
- r for one aspect
- t_7 for two aspects
- t_8 for all three aspects
• Independence argument used with correct probabilities to determine events are not independent.
• Indicates that the school's assumption is incorrect
• Describes the nature of the relationship between the two events.</td></tr>
</table>

(d)(i)	P(not intending on going to university and not doing this in year immediately after high school) = 0.031 $0.031 \times 475 = 14.725 = 15$ students Accept 14 students.	• Correct number of students calculated.		
(ii)	P(preferred | university) $= \frac{63.4}{71.6} = 0.89$ P(preferred | not university) $= \frac{25.3}{28.4} = 0.89$ The claim is not supported as P(preferred | university) is not twice. P(preferred | not university). *Accept "nearly equal" instead of "not twice".*	• At least one conditional probability is correct.	• Comparison of correct conditional probabilities AND Correct **justified** conclusion that the claim is not supported	
(e)	 P(entered for 91584 | entered for 91586) $= \frac{17}{72} = 0.2361$ $\frac{17}{72} \times 9985 = 2358$ students Accept 2357 students	• Venn diagram or table correct.	• Correct number of students found.	

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Making progress / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t_7	1 of t_8

Q	Expected Coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)(i)	For each year the proportion of males attending is as follows: $2017 = \frac{18\,168}{42\,302} = 0.4295$ $2018 = \frac{18\,192}{42\,717} = 0.42595$ $2019 = \frac{18\,296}{43\,042} = 0.4251$ $2020 = \frac{18\,065}{42\,825} = 0.4218$ 2017 is the year with the greatest proportion of males attending Auckland University.	• Correct proportions.	• Correct proportions and year determined and justified by comparison with probabilities from other years.	
(ii)	• The data given from 2017 to 2020 is "old", so may not still be applicable. • Part-time students may be improperly included in the data for each year. • 2020 and the start of Covid may have made tracking such data much harder, increasing the chances of error. • Over a period of 4 years the university may have changed the definitions of what it is to be an attending student. • Not all students enrolled with Auckland University will be attending. • Students dropping out during the year will affect proportions. • Students identifying themselves as male or female may change during a year. *Accept other valid reason(s).*	• ONE valid reason given.	• TWO valid reasons given.	

<table>
<tr><td>(b)(i)</td><td>P(attended lectures regularly) $= \frac{41}{50} \times 0.38 + \frac{9}{50} \times 0.21 = 0.3494$</td><td>• Correct probability.</td><td></td><td></td></tr>
<tr><td>(ii)</td><td>P(E ∩ A) + P(E′ ∩ A′) = 0.3116 + 0.0378 = 0.4538

The statement is not strictly correct and is not guaranteed to equal 1 since it is missing two events from the combinations of possible events to make it the full “suite” of complementary events to sum to 1.

<table><tr><td></td><td>A</td><td>A′</td><td></td></tr><tr><td>E</td><td>0.3116</td><td>0.5084</td><td>0.82</td></tr><tr><td>E′</td><td>0.0378</td><td>0.1422</td><td>0.18</td></tr><tr><td></td><td>0.3494</td><td>0.6506</td><td>1</td></tr></table></td><td>• Finds the sum of the two probabilities.</td><td>• A statement that the other two events have not been considered or are missing.</td><td></td></tr>
<tr><td>(iii)</td><td>P(one student took a course without final examination and attended lectures regularly) $= \frac{9}{50} \times 0.21 = 0.0378$

P(three of the four students) = $0.0378^3 \times 0.9622 \times 4 = 0.000208$ (3 sf)

We would need to assume that the number of students is sufficiently large so that selecting one does not change the probability for the remaining students.

The probabilities are not likely to be the same from year to year given factors which impact students, university destinations (such as economic factors, COVID, etc…). The university would need to look at other factors and build up a trend over several years of these associated probabilities.

Accept discussion about results from this university not necessarily being applicable to those in all of NZ.</td><td>• One student probability calculated correctly.

Accept 2.08×10^{-4}.

Accept use of binomial distribution.</td><td>• Three of four students’ probability calculated correctly.</td><td>• One student probability calculated correctly
AND
t_7 for 1 below t_8 for both below
- Assumption of independence is correctly described with sufficiently large number of students described.
- Correctly reasoned argument about risks of applying to 2022 intake of first-year students.</td></tr>
</table>

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Making progress / attempt at one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t_7	1 of t_8

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<tr><th>Q</th><th>Expected Coverage</th><th>Achievement (u)</th><th>Merit (r)</th><th>Excellence (t)</th></tr>
<tr><td>THREE
(a)(i)</td><td>$\frac{1}{10}$
Given no knowledge and the activity and testing the assumption that the ten digits are equally likely, the ‘best’ and probably only theoretical probability to select is $\frac{1}{10}$ or 0.1</td><td>• Probability of $\frac{1}{10}$ given.</td><td>• Valid Justification for $\frac{1}{10}$.</td><td></td></tr>
<tr><td>(ii)</td><td>$\text{P(5 or higher | less than 8)} = \frac{17}{59} = 0.288 \text{ (3 s. f.)}$</td><td>• Correct probability.</td><td></td><td></td></tr>
<tr><td>(b)(i)</td><td>$\text{P(digit 8 from student 2 sample)} = \frac{8}{60} = 0.133$
$\text{P(digit 8 from student 3 sample)} = \frac{7}{66} = 0.106$
The claim is not valid because the probabilities of seeing the digit 8 for students 2 and 3 are not too different from the expected theoretical probability (0.1).</td><td rowspan="2">• Probabilities for a digit 8 calculated for students 2 and 3.</td><td rowspan="2">• Comparison of the probabilities with 0.1 and saying the claim is not valid. Numerical evidence required.</td><td rowspan="2">• Comparison of the probabilities with 0.1 and saying the claim is not valid. Numerical evidence required
AND
Some correct notion that the visualisations are the result of a low sample size.</td></tr>
<tr><td>(ii)</td><td>From the first student’s visualisation together with the subsequent three visualisations are showing that randomness with low samples will produce large sampling variability that we can see within and between the visualisations.
This does not necessarily mean the numbers on standard number plates are not equally likely.</td></tr>
</table>

(c)	We can see that the proportion of the time that zero occurred in the sample [the **experimental probability**] is not 0.1 [the **model probability**]. We have evidence that the initial model probability of 1/10 for each digit is not the right model for the number plate digits, as shown by the likely variation from the simulation results. The **model probability** of 0.1 is attempting to model the unknown **true probability** which is likely to be less than 0.1.	• Indication that the proportion of the digit 0 occurring is not 0.1.	• Indication that the proportion of the digit 0 occurring is not 0.1 considering sampling variation, as seen from the results of the simulation.	• Indication that the proportion of the digit 0 occurring is not 0.1 considering sampling variation, as seen from the results of the simulation AND A correct reference to at least TWO of the probability types.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 12	13 – 18	19 – 24