Assessment Schedule – 2024

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$f(-3) = 0$ gives $-27 + 9p - 15 - 12 = 0$ $9p = 54$ $p = 6$	• Correct solution for p.		
(b)	$z^{15} = \left[m\operatorname{cis}\left(\frac{n\pi}{5}\right)\right]^{15}$ $z^{15} = m^{15}\operatorname{cis}\left(\frac{15n\pi}{5}\right)$ $z^{15} = m^{15}\operatorname{cis}(3n\pi)$	• Correct solution.		
(c)	$\left[4-\sqrt{kx}\right]^2 = \left[\sqrt{kx+4}\right]^2$ $(4-\sqrt{kx})(4-\sqrt{kx}) = kx+4$ $16-8\sqrt{kx}+kx = kx+4$ (1) $8\sqrt{kx} = 12$ $\sqrt{kx} = \frac{3}{2}$ $kx = \frac{9}{4}$ $x = \frac{9}{4k}$ Or equivalent.	• Reaching line (1).	• Correct value for x.	
(d)	Let $z = x + y\text{i}$ $\lvert x+y\text{i}-\text{i}\rvert = \lvert x+y\text{i}+1\rvert$ $\lvert x+(y-1)\text{i}\rvert = \lvert (x+1)+y\text{i}\rvert$ $\sqrt{x^2+(y-1)^2} = \sqrt{(x+1)^2+y^2}$ $x^2+(y-1)^2 = (x+1)^2+y^2$ (1) $x^2+y^2-2y+1 = x^2+2x+1+y^2$ $-2y = 2x$ $y = -1x$ i.e. Gradient will be –1. Accept a graphical method.	• Reaching line (1).	• Correct gradient found.	

(e)	$w=\dfrac{2+3k\text{i}}{4+5k\text{i}}$ $w=\dfrac{(2+3k\text{i})(4-5k\text{i})}{(4+5k\text{i})(4-5k\text{i})}$ $w=\dfrac{8-10k\text{i}+12k\text{i}-15k^2\text{i}^2}{16-20k\text{i}+20k\text{i}-25k^2\text{i}^2}$ $w=\dfrac{(8+15k^2)+2k\text{i}}{16+25k^2}$ (1) If w does lie on the line $y=x$ then: $\dfrac{2k}{16+25k^2}=\dfrac{8+15k^2}{16+25k^2}$ $2k=8+15k^2$ $0=15k^2-2k+8$ (2) Consider $b^2-4ac=(-2)^2-4\times15\times8$ $=4-480=-476<0$ i.e. As the discriminant is negative, there are no solutions to the equation. i.e. No values of k possible to satisfy the requirements for $y=x$. As required.	• Reaching line (1).	• Reaching line (2).	• t1 / E7 Correct conclusion, with incomplete justification. OR Complete solution with a minor error • t2 / E8 Correct conclusion, with valid and clear justification.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\frac{i(2k-i)}{(2k+i)(2k-i)} = \frac{2ki - i^2}{4k^2 - 2ki + 2ki - i^2}$ $= \frac{2ki+1}{4k^2+1} = \frac{1+2ki}{4k^2+1}$	• Correct solution.		
(b)	Equal roots $\Rightarrow b^2 - 4ac = 0$ $(3+2r)^2 - 4 \times 2 \times (3-2r) = 0$ $9 + 12r + 4r^2 - 24 + 16r = 0$ $4r^2 + 28r - 15 = 0$ $(2r-1)(2r+15) = 0$ $r = \frac{1}{2} = 0.5$ or $r = \frac{-15}{2} = -7.5$	• Correct solutions for both values of r.		
(c)	$w = (w+i)(2-i)$ $w = 2w - wi + 2i - i^2$ $w - 2w + wi = 2i + 1$ $-w + wi = 1 + 2i$ $w(-1+i) = 1 + 2i$ $w = \frac{1+2i}{-1+i}$ (1) $w = \frac{(1+2i)(-1-i)}{(-1+i)(-1-i)}$ $w = \frac{-1 - i - 2i - 2i^2}{1 + i - i - i^2}$ $w = \frac{1-3i}{2} = \frac{1}{2} - \frac{3}{2}i$ Then $\lvert w \rvert = \sqrt{\left(\frac{1}{2}\right)^2 + \left(\frac{-3}{2}\right)^2}$ $\lvert w \rvert = \sqrt{2.5} = \sqrt{\frac{5}{2}} = \frac{\sqrt{10}}{2} = 1.5811$ Alternative method: $w = (w+i)(2-i)$ $w = 2w - wi + 2i - i^2$ $w - 2w + wi = 2i + 1$ $-w + wi = 1 + 2i$ $w(-1+i) = 1 + 2i$ $w = \frac{1+2i}{-1+i}$ (1) $\lvert w \rvert = \frac{\lvert 1+2i \rvert}{\lvert -1+i \rvert}$ $\lvert w \rvert = \frac{\sqrt{1^2 + 2^2}}{\sqrt{(-1)^2 + 1^2}}$ $\lvert w \rvert = \frac{\sqrt{5}}{\sqrt{2}} = \sqrt{2.5} = \frac{\sqrt{10}}{2} = 1.5811$	Reaching line (1).	• Correct value for $\lvert w \rvert$ found.	

(d)	If $z_1 = 6-2i \Rightarrow z_2 = 6+2i$ Using $z = 6+2i \Rightarrow z-6 = 2i$ $(z-6)^2 = (2i)^2$ $z^2 - 12z + 36 = -4$ $z^2 - 12z + 40 = 0$ Then $f(z) = (Az+B)(z^2 - 12z + 40)$ $A = 2$ and $B = -5$ So $d = -29$ and $z_3 = \frac{5}{2}$	• Correct value of d. OR Solutions for z_2 and z_3 found correctly.	• Correct value of d. AND Solutions for z_2 and z_3 found correctly, with evidence of algebraic methods.	
(e)	Let $z = x + yi$ then $\|x + iy - 1 - 7i\| = 2\|x + yi - 4 - 4i\|$ $\|(x-1)+(y-7)i\| = 2\|(x-4)+(y-4)i\|$ $\sqrt{(x-1)^2+(y-7)^2} = 2\sqrt{(x-4)^2+(y-4)^2}$ $(x-1)^2+(y-7)^2 = 4\left[(x-4)^2+(y-4)^2\right]$ (1) $x^2 - 2x + 1 + y^2 - 14y + 49$ $= 4\left[x^2 - 8x + 16 + y^2 - 8y + 16\right]$ $x^2 - 2x + y^2 - 14y + 50$ $= 4x^2 - 32x + 4y^2 - 32y + 128$ $0 = 3x^2 - 30x + 3y^2 - 18y + 78$ (2) $0 = x^2 - 10x + y^2 - 6y + 26$ $0 = (x-5)^2 - 25 + (y-3)^2 - 9 + 26$ $(x-5)^2 + (y-3)^2 = 8$ Then $x = 3$ gives: $(3-5)^2 + (y-3)^2 = 8$ $4 + (y-3)^2 = 8$ $(y-3)^2 = 4$ $y - 3 = \pm 2$ $y = 5$ or $y = 1$ i.e. $u = 3+5i$ or $u = 3+i$	• Reaching line (1).	• Reaching line (2).	• t1 / E7 Finding the two values of y but not connecting these to the two values of u. OR Complete solution with a minor error. • t2 / E8 Correct solution, finding the equation of the locus in the required format AND both values of u.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\frac{\sqrt{2p}\left(\sqrt{2p}+\sqrt{p}\right)}{\left(\sqrt{2p}-\sqrt{p}\right)\left(\sqrt{2p}+\sqrt{p}\right)} = \frac{2p+\sqrt{2p^2}}{2p-p}$ $= \frac{2p+\sqrt{2}p}{p} = 2+\sqrt{2}$	• Correct solution.		
(b)	$z = -2+3\text{i}$ $z^2 = (-2+3\text{i})^2$ $z^2 = 4-12\text{i}-9$ $z^2 = -5-12\text{i}$	• –5 – 12i correctly plotted.		
(c)	$3-d\text{i} = \frac{10d}{3+d\text{i}}$ $(3-d\text{i})(3+d\text{i}) = 10d$ $9+3d\text{i}-3d\text{i}-d^2\text{i}^2 = 10d$ (1) $9-d^2\text{i}^2 = 10d$ $d^2-10d+9=0$ $(d-1)(d-9)=0$ $d=1 \text{ or } d=9$	• Reaching line (1).	• Finding both values of d, with justification.	
(d)	$z^4 = -81k^8$ $z^4 = 81k^8 cis\pi$ $\vartheta_1 = \frac{\pi}{4}, \vartheta_2 = \frac{3\pi}{4}$ $\vartheta_3 = \frac{5\pi}{4} = -\frac{3\pi}{4}, \vartheta_4 = \frac{7\pi}{4} = -\frac{\pi}{4}$ $z_1 = 3k^2\text{cis}\frac{\pi}{4}$ $z_2 = 3k^2\text{cis}\frac{3\pi}{4}$ $z_3 = 3k^2\text{cis}\frac{5\pi}{4} = 3k^2\text{cis}-\frac{3\pi}{4}$ $z_4 = 3k^2\text{cis}\frac{7\pi}{4} = 3k^2\text{cis}-\frac{\pi}{4}$	• One correct solution.	• Four correct solutions.	

(e)	$x+\frac{1}{x}=p$ $\left(x+\frac{1}{x}\right)^3=p^3$ $\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}\right)\left(x+\frac{1}{x}\right)=p^3$ $\left(x^2+2+\frac{1}{x^2}\right)\left(x+\frac{1}{x}\right)=p^3$ $x^3+x+2x+\frac{2}{x}+\frac{1}{x}+\frac{1}{x^3}=p^3$ (1) $x^3+\frac{1}{x^3}+3x+\frac{3}{x}=p^3$ $x^3+\frac{1}{x^3}+3\left(x+\frac{1}{x}\right)=p^3$ (2) $x^3+\frac{1}{x^3}+3p=p^3$ $x^3+\frac{1}{x^3}=p^3-3p$	• Reaching line (1).	• Reaching line (2).	• t1 / E7 Correct expression, but without including clear communication. OR Complete solution with a minor error. • t2 / E8 Correct expression, including clear communication.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 06	07 – 13	14 – 18	19 – 24