Assessment Schedule – 2024

Calculus: Apply differentiation methods in solving problems (91578)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$f'(x) = -18x^3(4-9x^4)^{-\frac{1}{2}}$ Or equivalent	• Correct derivative.		
(b)	$\frac{dy}{dx} = (2x+3)\sin x + (x^2+3x+2)\cos x$ $x = 0$ gives $\frac{dy}{dx} = 2$	• Correct gradient of tangent, with correct derivative.		
(c)	Decreasing, therefore $6(2x-7)\times 2 + \frac{60}{x} + 0 < 0$ (1) $24x - 84 + \frac{60}{x} < 0$ $24x^2 - 84x + 60 < 0$ $2x^2 - 7x + 5 < 0$ $(2x-5)(x-1) < 0$ $1 < x < 2.5$	• Evidence of reaching stage (1). Can set = 0 and find the correct critical values for (u).	• Finds the correct inequality for required values of x for decreasing function, with evidence of derivative.	
(d)	$\frac{dy}{dx} = e^{-2x}(4-4x)$ $\frac{d^2y}{dx^2} = e^{-2x}(8x-12)$ Solving $\frac{dy}{dx} = e^{-2x}(4-4x) = 0$ gives $x = 1$ Then $x = 1$ gives $\frac{d^2y}{dx^2} = -0.541 < 0$ i.e. Maximum when $x = 1$.	• Correct expression for $\frac{dy}{dx}$.	• Finds $x = 1$ and justifies that this is a maximum using a calculus method, and with evidence of derivative.	

(e)	$\frac{dy}{dx} = 2 + x^{-2} - \frac{2}{x}$ $\frac{d^2y}{dx^2} = -2x^{-3} + 2x^{-2}$ For an inflection, solve $\frac{d^2y}{dx^2} = -2x^{-3} + 2x^{-2} = 0$ $\frac{2}{x^2} = \frac{2}{x^3}$ $2x^3 = 2x^2$ $2x^3 - 2x^2 = 0$ $2x^2(x-1) = 0$ Either $x = 0$ ignore as not valid Or $x = 1$ $x = 1$ gives $y = 1$, i.e. $P = (1,1)$ $x = 1$ gives $\frac{dy}{dx} = 1$ Then equation of tangent is: $y - 1 = 1(x - 1)$ $y = x$	• Correct expression for $\frac{dy}{dx}$.	• Finds $x = 1$, with correct $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.	• E7 / T1 Consistent equation of tangent, from incorrect P. OR Correct solution but with one minor error. • E8 / T2 Correct equation of tangent found.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	T1	T2

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\frac{dx}{dt} = 6t$ $\frac{dy}{dt} = -\sin t$ $\frac{dy}{dx} = \frac{-\sin t}{6t}$	• Correct expression for $\frac{dy}{dx}$. • $\frac{dy}{dx}$ can be expressed in term of x, y.		
(b)	$\frac{ds}{dt} = v = \frac{6t+5}{3t^2+5t+2}$ $t = 1$ gives $v = \frac{11}{10} = 1.1 \text{ m sec}^{-1}$ Units not required.	• Correct derivative. AND Correct value for velocity of $v = \frac{11}{10} = 1.1$.		
(c)	$\frac{dy}{dx} = 2x.\cos(x^2) + \sin x$ $\frac{d^2y}{dx^2} = 2\cos(x^2) - 2x.2x\sin(x^2) + \cos x$ $\frac{d^2y}{dx^2} = 2\cos(x^2) - 4x^2\sin(x^2) + \cos x$ Then LHS $= \frac{d^2y}{dx^2} + 4x^2y$ $= 2\cos(x^2) - 4x^2\sin(x^2) + \cos x + 4x^2(\sin(x^2) - \cos x)$ $= 2\cos(x^2) + \cos x - 4x^2\cos x$ $= 2\cos(x^2) + (1-4x^2)\cos x$ $=$ RHS As required.	• Correct expression for $\frac{dy}{dx}$.	• Correct justification that it is a solution, with evidence of both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.	
(d)	$f'(x) = \frac{1-\ln x}{x^2}$ $f''(x) = \frac{-3+2\ln x}{x^3}$ For inflection $f''(x) = 0$ giving $-3 + 2\ln x = 0$ $\ln x = \frac{3}{2}$ $x = e^{1.5} = 4.4817$ $y = 1.5e^{-1.5} = 0.3347$ i.e. $(4.4817, 0.3347)$	• Correct expression for $\frac{dy}{dx}$.	• Correct co-ordinates for point of inflection, with evidence of both $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.	

(e)	$\frac{dy}{dx}=\frac{e^{3x}(6x^2+3kx+k)}{(2x+k)^2}$ For turning points $\frac{dy}{dx}=0$ $e^{3x}(6x^2+3kx+k)=0$ Either $e^{3x}=0$ No solutions Or $6x^2+3kx+k=0$ (a) But, as only a single turning point, using $b^2-4ac=0$ $(3k)^2-4\times6\times k=0$ $9k^2-24k=0$ $3k(3k-8)=0$ Either $k=0$ Ignore as not valid Or $3k-8=0$ i.e. $k=\frac{8}{3}$ Substituting $k=\frac{8}{3}$ into equation (a) gives $6x^2+8x+\frac{8}{3}=0$ $9x^2+12x+4=0$ $(3x+2)(3x+2)=0$ $x=-\frac{2}{3}$	• Correct $\frac{dy}{dx}$.	• Finds the correct value of k, with evidence of calculus methods.	• E7 / T1 Correct solution but with one minor error. • E8 / T2 Calculates x-co-ordinate of Q, with clear and full calculus justification.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	T1	T2

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\frac{dy}{dx} = \frac{1}{2}x^{-\frac{1}{2}}\sec 6x + 6x^{\frac{1}{2}}\sec 6x \tan 6x$ Or equivalent.	• Correct derivative.		
(b)(i) (ii) (iii)	$x = 5$ $x = 1$ and $3 < x < 5$ 1	• Two out of three correct responses.		
(c)	$f'(x) = \frac{10x^2 - 40}{(x^2 + 5x + 4)^2}$ For stationary points, $f'(x) = 0$ $10x^2 - 40 = 0$ $x = 2$ or $x = -2$	• Correct expression for $f'(x)$.	• x-values of both stationary points found, with evidence of a calculus method.	
(d)	Method 1: $V = \frac{1}{3}\pi r^2 h = \frac{2}{3}\pi r^3$ $\frac{dV}{dr} = 2\pi r^2$ We require $\frac{dh}{dt} = \frac{dh}{dr} \times \frac{dr}{dV} \times \frac{dV}{dt}$ $\frac{dh}{dt} = 2 \times \frac{1}{2\pi r^2} \times 3$ $= \frac{3}{4\pi} = 0.2387 \text{ cm sec}^{-1}$ OR Method 2: $V = \frac{1}{3}\pi r^2 h = \frac{1}{12}\pi h^3$ $\frac{dV}{dh} = \frac{1}{4}\pi h^2$ We require $\frac{dh}{dt} = \frac{dh}{dV} \times \frac{dV}{dt}$ $\frac{dh}{dt} = \frac{4}{\pi h^2} \times 3$ $= \frac{3}{4\pi} = 0.2387 \text{ cm sec}^{-1}$ Units not required.	• Correct expression for $\frac{dh}{dt}$.	• Correct value for $\frac{dh}{dt}$, with evidence of a calculus method.	

(e)	Let P have coordinates (x, y) Area of triangle $OPQ = xe^{-x^2}$ $\frac{\mathrm{d}A}{\mathrm{d}x} = e^{-x^2}(1-2x^2)$ For max / min, $\frac{dA}{dx} = 0$ $e^{-x^2}(1-2x^2) = 0$ Either $e^{-x^2} = 0$ No solutions Or $(1-2x^2) = 0$ i.e. $x = \pm\frac{1}{\sqrt{2}} = \pm 0.7071$ But ignore $x = -\frac{1}{\sqrt{2}}$ as $x > 0$ Then area $= \frac{1}{\sqrt{2}} \times e^{-\frac{1}{2}} = \frac{1}{\sqrt{2}} \times \frac{1}{e^{\frac{1}{2}}}$ Area $= \frac{1}{\sqrt{2e}}$ as required.	• Correct expression for $\frac{\mathrm{d}A}{\mathrm{d}x}$.	• Finding $x = \frac{1}{\sqrt{2}}$, and evidence of ignoring $x = -\frac{1}{\sqrt{2}}$ with evidence of a calculus method.	• E7 / T1 Correct proof, but with one minor error. • E8 / T2 Correct proof of exact area value, with evidence of a calculus method.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	T1	T2

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 07	08– 12	13 – 18	19– 24