Assessment Schedule – 2024

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
ONE (a)	$3\sec 2x + c$	• Correct integral $+\,c$ not required.		
(b)	Area $= \int_{-0.4}^{0} 40x\left(5x^2-3\right)^3 \mathrm{d}x$ $= \left[\left(5x^2-3\right)^4\right]_{-0.4}^{0}$ $= \left[81 - 23.4256\right]$ $= 57.5744 \text{ units}^2$	• Correct solution, with evidence of correct integration. *Units not required.*		
(c)	$d = \int 26.4t^{\frac{1}{3}}\,\mathrm{d}t$ $d = \dfrac{26.4t^{\frac{4}{3}}}{\frac{4}{3}} + c$ $d = 19.8t^{\frac{4}{3}} + c$ When $t = 0$ and $d = 360$ gives $c = 360$. i.e. $d = 19.8t^{\frac{4}{3}} + 360$ (1) When $v = 264$ gives $t = 1000$ When $t = 1000$ gives $d = 198\,360$ m.	• Reaching stage (1) of the solution.	• Correct solution, with evidence of correct integration. *Accept any reasonable rounding.* *Units not required.*	
(d)	$y = \int 24\cos x \, 3x \sin x \, \mathrm{d}x$ $y = 12\int\left(\sin 4x - \sin 2x\right)\mathrm{d}x$ $y = 12\left[-\frac{1}{4}\cos 4x + \frac{1}{2}\cos 2x\right] + c$ $y = -3\cos 4x + 6\cos 2x + c$ When $x = \frac{\pi}{3}$ and $y = 6$ gives $c = 7.5$ *i.e.* $y = -3\cos 4x + 6\cos 2x + 7.5$ When $x = \frac{\pi}{2}$ gives $y = -1.5$	• Correct integration.	• Correct solution, with evidence of correct integration.	

(e)	$A=\int_0^1 3\sec^2 x\,\mathrm{d}x-\int_0^1 2\tan^2 x\,\mathrm{d}x$ $A=3\left[\tan x\right]_0^1-2\int_0^1 \sec^2 x-1\,\mathrm{d}x$ $A=3\left[\tan 1-\tan 0\right]-2\left[\tan x-x\right]_0^1$ $A=3\left[\tan 1-0\right]-2\left[\tan 1-1-0\right]$ $A=3\tan 1-2\tan 1+2$ $A=\tan 1+2$ $A=3.557\text{ units}^2$ *Units not required.* *Accept any reasonable rounding.*	• For either integration correct.	• For both integrations correct.	• E7 Solution with minor error. • E8 Correct solution, with evidence of correct integrations.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
TWO (a)	$\int\left(9x^8+24x^4+16\right)\mathrm{d}x$ $=x^9+\frac{24x^5}{5}+16x+c$	• Correct integral. $+\,c$ not required.		
(b)	$3\int_k^{16} x^{\frac{1}{2}}\,\mathrm{d}x=112$ $3\left[\frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right]_k^{16}=112$ $2\left[x^{\frac{3}{2}}\right]_k^{16}=112$ $64-k^{\frac{3}{2}}=56$ $k^{\frac{3}{2}}=8$ $k=4$	• Correct solution, with evidence of correct integration.		
(c)	$\int\frac{1}{y^2}\,\mathrm{d}y=\int 12\mathrm{e}^{3x}\mathrm{d}x$ $-\frac{1}{y}=4\mathrm{e}^{3x}+c$ (1) $y=0.5$ and $x=0$ give $c=-6$ i.e. $-\frac{1}{y}=4\mathrm{e}^{3x}-6$ When $x=\frac{1}{3}$ gives $y=\frac{-1}{4e-6}\approx-0.2052$	• Reaching stage (1) of the solution.	• Correct solution, with evidence of correct integration. *Accept any reasonable rounding.*	
(d)	Area = $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(1-\sin^2 x\right)\mathrm{d}x$ $=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(1-\left(\tfrac{1}{2}-\tfrac{1}{2}\cos 2x\right)\right)\mathrm{d}x$ $=\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(\tfrac{1}{2}+\tfrac{1}{2}\cos 2x\right)\mathrm{d}x$ $=\left[\tfrac{1}{2}x+\tfrac{1}{4}\sin 2x\right]_{-\frac{\pi}{2}}^{\frac{\pi}{2}}$ (1) $=\frac{\pi}{2}$	• Reaching stage (1) of the solution.	• Correct solution, with evidence of correct integration. *Accept any reasonable rounding.* *Units not required.*	

(e)	$M=\dfrac{4\pi a}{3b}\displaystyle\int_0^p 3br^2\left(\dfrac{1}{1+br^3}\right)\mathrm{d}r$ $=\dfrac{4\pi a}{3b}\left[\ln\left(1+br^3\right)\right]_0^p$ (1) $=\dfrac{4\pi a}{3b}\left[\ln\left(1+bp^3\right)-\ln 1\right]$ $=\dfrac{4\pi a}{3b}\ln\left(1+bp^3\right)$		• Reaching stage (1) of the solution.	• E7 Solution with minor error. • E8 Correct solution, with evidence of correct integration.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

	Expected coverage	Achievement (u)	Merit (r)	Excellence (t)
THREE (a)	$\frac{1}{2}e^{2x} - \frac{3}{4}e^{-4x} + c$	• Correct integral $+\ c$ not required.		
(b)	$y = \int \frac{5}{4x-3} dx$ $y = \frac{5}{4}\ln\|4x-3\| + c$ $y = 10$ and $x = 1$ gives $c = 10$ i.e. $y = \frac{5}{4}\ln\|4x-3\| + 10$	• Correct solution, with evidence of correct integration. *Accept any reasonable rounding.*		
(c)	$\frac{4x+5}{2x+3} = \frac{4x+6-1}{2x+3} = 2 - \frac{1}{2x+3}$ OR by long division or by inspection. $\int_{-1}^{m}\left(2 - \frac{1}{2x+3}\right)dx = 2m$ $\left[2x - \frac{1}{2}\ln\|2x+3\|\right]_{-1}^{m} = 2m$ (1) $\left[2m - \frac{1}{2}\ln(2m+3)\right] - \left[-2 - \frac{1}{2}\ln 1\right] = 2m$ $2m - \frac{1}{2}\ln(2m+3) + 2 + 0 = 2m$ $4 = \ln(2m+3)$ $2m + 3 = e^4$ $m = \frac{e^4 - 3}{2} = 25.8$	• Reaching stage (1) of the solution.	• Correct solution, with evidence of correct integration. *Accept any reasonable rounding.*	

(d)	Area A = Area B $\int_0^k 2\cos\left(\frac{x}{2}\right)\mathrm{d}x = \int_k^\pi 2\cos\left(\frac{x}{2}\right)\mathrm{d}x$ $4\left[\sin\left(\frac{x}{2}\right)\right]_0^k = 4\left[\sin\left(\frac{x}{2}\right)\right]_k^\pi$ $\sin\left(\frac{k}{2}\right) - 0 = \sin\left(\frac{\pi}{2}\right) - \sin\left(\frac{k}{2}\right)$ $2\sin\left(\frac{k}{2}\right) = 1$ $\sin\left(\frac{k}{2}\right) = \frac{1}{2}$ $\frac{k}{2} = \frac{\pi}{6}$ Accept $\frac{k}{2} = 0.5236$ Do not accept $\frac{k}{2} = 30$ $k = \frac{\pi}{3} = 1.047$ **Alternative method:** Whole area $= \int_0^\pi 2\cos\left(\frac{x}{2}\right)\mathrm{d}x$ $= 4\left[\sin\left(\frac{x}{2}\right)\right]_0^\pi$ $= 4(1-0) = 4$ Then Area A = Area B $= 2$ i.e. $\int_0^k 2\cos\left(\frac{x}{2}\right)\mathrm{d}x = 2$ $\Rightarrow 4\left[\sin\left(\frac{x}{2}\right)\right]_0^k = 2$ $\Rightarrow \sin\left(\frac{k}{2}\right) = \frac{1}{2}$ $\Rightarrow k = \frac{\pi}{3} = 1.047$	• Correct integration.	• Correct solution, with evidence of correct integration. *Accept any reasonable rounding.*	

(e)	$\frac{dN}{dt} = k(N-18)$ $\int \frac{1}{N-18} dN = \int k\,dt$ $\ln\lvert N-18\rvert = kt + c$ (1) $t = 30$ and $N = 50$ gives (using t in minutes) $\ln(50-18) = 30k + c$ $\ln 32 = 30k + c$ (a) $t = 90$ and $N = 30$ gives $\ln(30-18) = 90k + c$ $\ln 12 = 90k + c$ (b) Solving equations (a) and (b) simultaneously gives: k = -0.0163 and c = 3.95615 So $\ln\lvert N-18\rvert = -0.0163t + 3.95615$ When $t = 0$ gives: $\ln\lvert N-18\rvert = 0 + 3.95615$ $N - 18 = e^{3.95615}$ $N - 18 = 52.26$ $N = 70.26$ °C **Alternative method** (using t in hours) $t = 0.5$ and $N = 50$ gives $\ln(50-18) = 0.5k + c$ $\Rightarrow \ln 32 = 0.5k + c$ (a) $t = 1.5$ and $N = 30$ gives $\ln(30-18) = 1.5k + c$ $\Rightarrow \ln 12 = 1.5k + c$ (b) Solving equations (a) and (b) simultaneously gives $k = -0.9808$ $c = 3.95615$ So $\ln\lvert N-18\rvert = -0.9808t + 3.95615$ When $t = 0$ gives $N = 70.26$ °C	• Reaching stage (1) of the solution.	• Finding the values for k and c, with evidence of correct integration. *Accept any reasonable rounding.*	• **E7** Solution with minor error. • **E8** Correct solution, with evidence of correct integration. *Accept any reasonable rounding.*

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t with minor error(s).	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 06	07 – 12	13– 19	20 – 24