Assessment Schedule – 2024

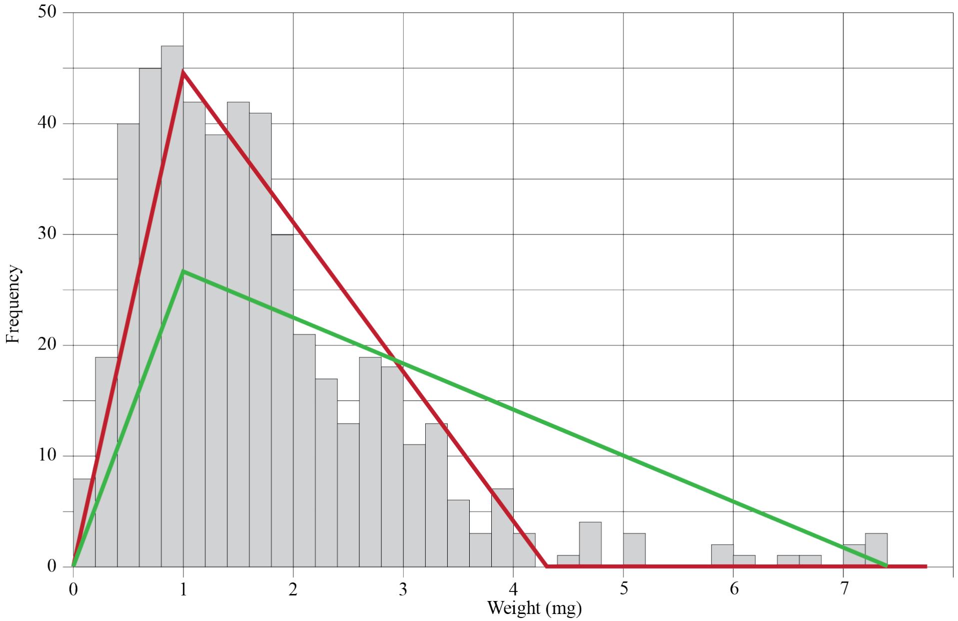
Mathematics and Statistics (Statistics): Apply probability distributions in solving problems (91586)

Evidence Statement

Q	Expected coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
ONE (a)(i)	Binomial distribution, $n = 30$, $p = 0.04$ P(X = 0) = 0.293858	• Probability correct. AND Distribution and parameters stated.		• Probability and parameters correct.
(ii)	The Binomial Distribution requires that: • There are only two outcomes – in this case students have an outie or an innie belly button. • There is a fixed number of trials – in this case the number of students is fixed (30 students). • The trials are independent of each other - The presence of an outie in one student does not affect the probability that other students have an outie (independence). • The probability of success remains constant - The probability of any student having an outie is constant (0.04).	• ONE correct reason identified in context.	• Probability and parameters correct AND TWO correct reasons identified in context.	AND
(iii)	Probability that one student has an outie: Binomial distribution, $n = 30$, $p = 0.04$ P(X = 1) = 0.36732 Probability that one student has an outie and there is belly button fluff: 0.36732×0.66 $= 0.24243$	• Probability for one student with an outie correct.	• Correct probability with working shown.	Probability for (a)(iii) correct.

(b)(i)	Normal distribution $\mu = 2.05$ cm $\sigma = 0.9$ cm P(1.5 < X < 2.0) = 0.2073 P(1.75 < X < 2.0) = 0.1084 P(X > 1.75 | 1 .5 < X < 2.0) $= \frac{0.1084}{0.2073} = 0.5229$	• Correct calculation of ONE normal probability.	• Correct conditional probability. with working shown. OR	• Correct conditional probability. with working shown. AND
(ii)	With the model parameters, the probability of having a negative navel diameter is about 1%, but negative diameters are not possible in the real world. P(X ≤ 0) = 0.0114 Either Since this is a pretty small percentage the normal model is appropriate Or So the normal model may not be appropriate *Note: It is possible for people to have no belly button.* *Alternative coverage:* Using the normal model suggests that 99% of people would have belly buttons within 3sd of the mean 2.05 ±(0.9 · 3), i.e. between –0.65 and 4.75 cm. 4.75 cm seems a very large belly button so the normal model does not seem appropriate *Accept other arguments supported by statistical evidence.*		• Reasoned discussion on suitability of model parameters in terms of the context.	• Reasoned discussion on suitability of model parameters in terms of the context.

NØ	**N1**	**N2**	**A3**	**A4**	**M5**	**M6**	**E7**	**E8**
No response; no relevant evidence.	Attempts at least one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

<table>
<tr><th>Q</th><th>Expected coverage</th><th>Achievement (u)</th><th>Achievement with Merit (r)</th><th>Achievement with Excellence (t)</th></tr>
<tr><td>TWO (a)(i)</td><td>
</td><td>• Green line correct.</td><td></td><td></td></tr>
<tr><td>(ii)</td><td>Height at X = 1.5
$h = \dfrac{2(7.4-1.5)}{(7.4-0)(7.4-1.0)} = 0.2492$
$P(X>1.5) = \dfrac{1}{2}\times(7.4-1.5)\times 0.2492$
$= \dfrac{1}{2}\times 5.9\times 0.2492$
$= 0.7350$
$P(X<1.5) = 1 - P(X>1.5)$
$= 1 - 0.7350$
$= 0.2650$</td><td>• Correct calculation of height at X = 1.5.</td><td>• Probability correctly calculated P(X < 1 .5).</td><td></td></tr>
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(iii)	Accept parameters within the following ranges: Minimum: a = 0 Maximum: $4 \leq b \leq 5.2$ Mode: $0.8 \leq c \leq 1.0$ For example: min = 0, max=5.4 and mode = 0.9 mg. The graph of the distribution of weights of belly button fluff has a minimum weight of zero (no fluff at all). Since it's not possible to have a negative weight of belly button fluff, and on the graph they had 0mg of fluff around 8 times I will keep the minimum at 0. There are only a few belly button fluff weights above 5 mg – these can be treated as outliers. There is a gap in the data after that point, so a maximum of around 5.2 would be a better fit for this data. The largest frequency of belly button fluff weights (mode) is between 0.8 mg and 1.0 mg. The proposed mode of 1mg seems to be at the top end of where most of the data is. A mode of 0.9 would be a better fit for the data. *Accept other valid justifications.*	• THREE of the parameters correct.	• THREE parameters correct. AND ONE Parameter with justification.	• THREE parameters correct. AND TWO parameters correct with justification.
(b)(i) (ii) (iii)	Mean = 2.031 A resident of this city can be expected to use, on average, about 2 towels on the one day. SD(T) = 1.405 SD(C) is 0.5, which is less than 1.405. Because the maximum number of clothing changes on this one day is only 3, compared to 6 for towel usage, the number of clothing changes is more tightly distributed about the mean, and the standard deviation is lower. If independent, then VAR(T + C) = VAR(T) + VAR(C) VAR(T + C) = 3.077 VAR(T) + VAR(C) $= 1.405^2 + 0.5^2$ = 1.974 + 0.25 =2.224 3.044 > 2.224 VAR(T + C) is greater than VAR(T) + VAR(C) Therefore T and C are not independent. This implies there is a link between variables T and C and suggests that they are dependent on each other. This is true, as people who shower more use more towels also have a clothing change.	• Correct answer. • Correct SD(T).	 • Correct SD(T) with contextual discussion linked to variation. • Correct conclusion supported by clear mathematical evidence.	 • Correct conclusion supported by clear mathematical evidence. AND Explained dependence of events in terms of context.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	Attempts at least one part of the question.	1 of u	2 of u	3 of u	1 of r	2 of r	1 of t	2 of t

Q	Expected coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
THREE (a)	Only 33 people out of 138 washed their belly button at least 7 times a week (an average of once per day). The proportion washing their belly button at least daily is about 33/138 = 0.239, which is less than one quarter. The headline is correct.	• Correct conclusion with numerical support.		
(b)(i)	Poisson $\lambda = \frac{2.8}{7} = 0.4$ $P(X \geq 1) = 1 - P(X = 0)$ $= 1 - 0.67032$ $= 0.32968$	• Correct lambda parameter. OR Correct calculation using $\lambda = 2.8$.	• Correct probability.	
(b)(ii)	The Poisson model assumes that the rate is proportional to the interval. It is assumed that the rate of belly button washes per week is scalable to give a rate of belly button washes per day – this may not be appropriate in this case, as there are many people who never wash their belly buttons. The Poisson model assumes that the event is random. Belly-button washing would not occur at random. It would be part of a regular washing regime, e.g. people might wash their belly button every second shower so it is predictable when they wash their belly button. The Poisson model assumes that the events are independent. If a person had a regular body washing routine which included washing their belly button, then when they washed their belly button could be affected by when they last washed their belly button so they would not be independent. *Accept other responses, e.g. the Poisson distribution assumes that the rate of belly-button washing per week remains constant. This may not be the case as some weeks people might wash them more often than others for example in summer when they may be hotter or if they have a job that makes them sweat more.* *Do not accept simultaneous arguments.*		• Assumption identified AND Explanation of why it is invalid in context.	

(iii)	Each grey line shows the expected outcome of one simulation of 138 randomly generated values from a Poisson distribution model with $\lambda = 2.8$. The grey band is made up of 1000 runs of these simulated results. This shows visually the sampling variation that can occur in the model.	• Grey band identified as the expected outcomes and frequencies when modelling the data using a Poisson distribution with lambda 2.8	• Grey band identified as the expected outcomes and frequencies when modelling the data using a Poisson distribution with lambda 2.8. AND How much variation to expect for the frequencies under this model when using (random) samples of size 138.	
(iv)	The simulation shows evidence that the proposed Poisson model is not appropriate to model the number of belly button washes per week. Each number of washes is not appearing in the frequency where variation due to natural variation can explain any discrepancies, as they are not "hidden" in the grey cloud of uncertainty. There is an excess frequency of 'zeros' and 'sevens' – a large proportion of people wash their belly buttons on average once per day, or never! The Poisson distribution does not model this aspect of the observed data.	• Comment that the proposed Poisson model is NOT appropriate supported by one reason.	• Comment that the proposed Poisson model is NOT appropriate supported by one reason that: EITHER Compares visually the frequencies of the real sample data with the tracked over-fitted shape. OR References sampling variation or what we would expect to see 'by chance alone'.	• Comment that the proposed Poisson model is NOT appropriate supported by: • Discussions that compare visually the frequencies of the real sample data with the tracked over-fitted shape. AND references sampling variation or what we would expect to see 'by chance alone'.

(v)	**Uniform distribution** a = 0 The minimum number of belly button washes is zero. $7 \leq b \leq 9$ There is no data for more than 9 belly button washes per week. A uniform model could be appropriate because there is a clear minimum and maximum number of belly button washes per week. Although the observed data does not appear to be uniformly distributed, taking sampling variation into account, it is possible that the true distribution of belly button washes per week is uniform. The uniform model assumes that all probabilities are equally likely so the frequencies of each number of belly button washings would be similar. In the observed data the frequency of 1 to 5 belly button washes is reasonably similar. So, the uniform model is less likely to overpredict the frequency of these belly button washes, unlike the current model which has significantly overpredicted them. A uniform model may not fit the frequency of the number of 0 and 7 belly button washes, however.	• Suggests Uniform model and identifies both parameters.	• Suggests Uniform model. AND Both parameters correct with justification.	• Suggests Uniform model. AND Both parameters correct with justification. AND Discussion of why uniform model fits the context.
	Alternative solution Identifies that none of the theoretical models could be used to model this data. Uses a model based on combinations of triangles and / or rectangles OR uses a discrete model based on the relative frequencies of the observed data (as below).	• Identifies that no standard theoretical model would fit the observed data.	• Identifies that a model could be based on a combination of shapes. OR • That a model could be based directly on experimental data	• Proposes a reasonable model either based on a combination of shapes (with comment about using a continuous model for discrete data) OR Creates a discrete model as below, using relative frequencies.

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Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0–7	8–13	14–18	19–24