Assessment Schedule – 2025

Calculus: Apply the algebra of complex numbers in solving problems (91577)

Evidence Statement

	Expected coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
ONE (a)	$f(x) = 21$ $48 + 8p - 8 + 5 = 21$ $8p = -24$ $p = -3$	• Correct value for p.		
(b)	$x = \dfrac{6k \pm \sqrt{36k^2 + 4 \times 1 \times k^2}}{2}$ $= \dfrac{6k \pm \sqrt{40k^2}}{2}$ $= \dfrac{6k \pm \sqrt{40}\,k}{2}$ $= 3k \pm \sqrt{10}\,k$	• Both solutions for x, in fully simplified form.		
(c)	$k^2x^2 = 3 - \dfrac{x}{k}$ $k^2x^2 + \dfrac{x}{k} - 3 = 0$ Consider $b^2 - 4ac$ $= \left(\dfrac{1}{k}\right)^2 - 4 \times k^2 \times -3$ $= \dfrac{1}{k^2} + 12k^2 > 0$ (1) This expression is always greater than 0 for all k, as k^2 is always positive, therefore the roots are always real. As required. Alternative Method : $k^2x^2 + \dfrac{x}{k} - 3 = 0$ $k^3x^2 + x - 3k = 0$ Consider $b^2 - 4ac$ $= 1 - 4 \times k^3 \times -3k$ $= 1 + 12k^4 > 0$ (1) This expression is always greater than 0 for all k, as k^4 is always positive, therefore the roots are always real. As required.	• Reaching stage (1) but without justification to explain the required proof.	• Clearly justified proof.	

(d)	$z^3 = -8m^{27}\mathrm{i} = 8m^{27}\mathrm{cis}\left(\frac{-\pi}{2}\right)$ $\varphi = \dfrac{-\frac{\pi}{2} + 2p\pi}{3}$ $p = 0 \Rightarrow \varphi_1 = -\frac{\pi}{6} = \frac{11\pi}{6}$ $p = 1 \Rightarrow \varphi_2 = \frac{\pi}{2}$ $p = 2 \Rightarrow \varphi_3 = \frac{7\pi}{6} = -\frac{5\pi}{6}$ $r = \sqrt[3]{8m^{27}} = 2m^9$ $z_1 = 2m^9\mathrm{cis}\left(-\frac{\pi}{6}\right) = 2m^9\mathrm{cis}\left(\frac{11\pi}{6}\right)$ $z_2 = 2m^9\mathrm{cis}\left(\frac{\pi}{2}\right)$ $z_3 = 2m^9\mathrm{cis}\left(\frac{7\pi}{6}\right) = 2m^9\mathrm{cis}\left(-\frac{5\pi}{6}\right)$	• One correct solution.	• All three correct solutions, with appropriate justification.	
(e)	$\lvert x + y\mathrm{i} - 5\mathrm{i} \rvert - \lvert x + y\mathrm{i} + 5\mathrm{i} \rvert = 4$ $\lvert x + (y-5)\mathrm{i} \rvert = 4 + \lvert x + (y+5)\mathrm{i} \rvert$ $\sqrt{x^2 + (y-5)^2} = 4 + \sqrt{x^2 + (y+5)^2}$ $x^2 + (y-5)^2 = 16 + x^2 + (y+5)^2 + 8\sqrt{x^2 + (y+5)^2}$ (1) $x^2 + y^2 - 10y + 25 = 16 + x^2 + y^2 + 10y + 25 + 8\sqrt{x^2 + (y+5)^2}$ $-20y - 16 = 8\sqrt{x^2 + (y+5)^2}$ $-5y - 4 = 2\sqrt{x^2 + y^2 + 10y + 25}$ $25y^2 + 16 + 40y = 4\left(x^2 + y^2 + 10y + 25\right)$ $25y^2 + 16 + 40y = 4x^2 + 4y^2 + 40y + 100$ $21y^2 - 4x^2 = 84$ Accept other fully simplified solutions.	• Reaching stage (1). OR Equivalent.	• Eliminating the square roots, on the way towards finding the equation of the locus.	**E7** Equation of locus found, with clear justification, but with one minor error. **E8** Equation of locus found, with clear justification.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t (minor error)	1t

	Expected coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
TWO (a)	$z = 2(4+2\text{i}) + 3(-2-3\text{i})$ $z = 8 + 4\text{i} - 6 - 9\text{i}$ $z = 2 - 5\text{i}$	• Correct value of z found. AND Plotted on the Argand diagram.		
(b)	$u^5 = m^5 \text{cis}\left(\frac{5 \times 3\pi}{10}\right)$ $u^5 = m^5 \text{cis}(1.5\pi)$ $u^5 = m^5(\cos 1.5\pi + \text{i} \sin 1.5\pi)$ $u^5 = m^5(0 - 1\text{i})$ $u^5 = -m^5\text{i}$	• Correct value of $u^5 = -m^5\text{i}$ found.		
(c)	$\sqrt{5^2 + (-m)^2} = \sqrt{5m^2}$ $25 + m^2 = 5m^2$ (1) $25 = 4m^2$ $m^2 = \frac{25}{4}$ $m = \pm\frac{5}{2} = \pm 2.50$ Must have both values of m.	• Reaching stage (1).	• Correct values of m found.	
(d)	$3g + gh\text{i} + 6\text{i} - 2h = 30 - 10\text{i} - 12\text{i} - 4$ Comparing real components gives $3g - 2h = 26$ $g = \frac{26 + 2h}{3}$ Comparing imaginary components gives $gh + 6 = -22$ $h\left(\frac{26 + 2h}{3}\right) + 6 = -22$ $26h + 2h^2 + 18 = -66$ $2h^2 + 26h + 84 = 0$ $h = -6$ and $h = -7$ $g = \frac{14}{3}$ and $g = 4$	• Establishing the pair of equations for g and h, by comparing real and imaginary components.	• Finding the two pairs of solutions for g and h.	

(e)	$\frac{d+6\text{i}}{1-d\text{i}} = \frac{(d+6\text{i})(1+d\text{i})}{(1-d\text{i})(1+d\text{i})}$ $= \frac{d+d^2\text{i}+6\text{i}+6d\text{i}^2}{1-d^2\text{i}^2}$ (1) $= \frac{-5d+(6+d^2)\text{i}}{1+d^2}$ If $\arg z = \frac{\pi}{4} \Rightarrow$ Real part = Imaginary part i.e. $\frac{-5d}{1+d^2} = \frac{6+d^2}{1+d^2}$ (2) $\Rightarrow -5d = 6+d^2$ $\Rightarrow d^2+5d+6=0$ $\Rightarrow (d+2)(d+3)=0$ $\Rightarrow d=-2$ or $d=-3$	• Reaching stage (1).	• Reaching stage (2).	**E7** Clear justification to find only one value of *d*. OR Correct solution but with one minor error. **E8** Clear justification to find both values of *d*.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t (minor error)	1t

	Expected coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
THREE (a)	$x-2=\sqrt{p}\,\mathrm{i}$ $x^2-4x+4=-p$ $x^2-4x+4+p=0$ Alternative method $x_1=2+\sqrt{p}\,\mathrm{i}$ and $x_2=2-\sqrt{p}\,\mathrm{i}$ $\left[x-\left(2+\sqrt{p}\,\mathrm{i}\right)\right]\left[x-\left(2-\sqrt{p}\,\mathrm{i}\right)\right]=0$ $x^2-\left(2-\sqrt{p}\,\mathrm{i}\right)x-\left(2+\sqrt{p}\,\mathrm{i}\right)x+\left(2+\sqrt{p}\,\mathrm{i}\right)\left(2-\sqrt{p}\,\mathrm{i}\right)=0$ $x^2-4x+4+p=0$	• Finding the correct quadratic equation.		
(b)	$=\left(\sqrt{3a}-\sqrt{12a}\,\mathrm{i}\right)\left(\sqrt{3a}-\sqrt{12a}\,\mathrm{i}\right)$ $=3a-6a\mathrm{i}-6a\mathrm{i}+12a\mathrm{i}^2$ $=-9a-12a\mathrm{i}$	• Simplifying the expression fully.		
(c)	$3+2\sqrt{x}+6\sqrt{x}+4x=5+6\sqrt{x}$ $3+8\sqrt{x}+4x=5+6\sqrt{x}$ $2\sqrt{x}=2-4x$ (1) $\sqrt{x}=1-2x$ $x=(1-2x)^2$ $x=1-4x+4x^2$ $4x^2-5x+1=0$ $(4x-1)(x-1)=0$ $x=\frac{1}{4}$ or $x=1$ But, after checking validity, there is only one solution of $x=\frac{1}{4}$	• Reaching stage (1).	• Both values of x found, but then $x=1$ is eliminated.	
(d)	$z_1=1+\mathrm{i}\Rightarrow z_2=1-\mathrm{i}$ $z=1+\mathrm{i}\Rightarrow z-1=\mathrm{i}$ $\Rightarrow z^2-2z+1=-1$ $\Rightarrow z^2-2z+2=0$ i.e. $f(z)=(Az+B)(z^2-2z+2)$ $\Rightarrow A=3$ and $B=-4$ i.e. $f(z)=(3z-4)(z^2-2z+2)=0$ Comparing co-efficient of z^2 gives $p=-6-4=-10$ Comparing co-efficient of z gives $q=6+8=14$ Therefore $z_1=1+\mathrm{i}$ $z_2=1-\mathrm{i}$ $z_3=\frac{4}{3}$	• The other two solutions found. OR Either p or q found.	• The other two solutions found. AND Both p and q found, with evidence of a fully justified algebraic method.	

(e)	$ui + 2v = 3$ (1) $u + (1-i)v = 4$ (2) Multiplying equation (2) by i gives: $ui + 2v = 3$ $ui + i(1-i)v = 4i$ Subtracting gives: $2v - i(1-i)v = 3 - 4i$ $2v - vi - v = 3 - 4i$ $v - vi = 3 - 4i$ $v(1-i) = 3 - 4i$ $v = \frac{3-4i}{1-i}$ $v = \frac{3-4i}{1-i} \times \frac{1+i}{1+i}$ $v = \frac{3+3i-4i+4}{1+1}$ $v = \frac{7-i}{2} = \frac{7}{2} - \frac{1}{2}i$ Then substituting v into equation (1) gives $ui + 2\left(\frac{7}{2} - \frac{1}{2}i\right) = 3$ $ui + 7 - i = 3$ $ui = -4 + i$ Multiplying by i gives: $-u = -4i - 1$ $u = 1 + 4i$ Allow alternative valid methods.	• Finding an equation involving one variable only. OR **CAO.**	• Finding solution for u or v.	**E** 7 Correct solution but with one minor error. **E 8** Finding solutions for both u and v.

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No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t (minor error)	1t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 12	13 – 19	20 – 24