Assessment Schedule – 2025

Calculus: Apply integration methods in solving problems (91579)

Evidence Statement

	Expected coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
ONE (a)	$2\sec 3x + c$	• Correct answer.		
(b)	$y = 2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} + c$ $c = -14$ $y = 2x^{\frac{3}{2}} + 4x^{\frac{1}{2}} - 14$	• Correct solution, with evidence of correct integration.		
(c)	$\int_1^2 (a - 6kx^{-2})\,\mathrm{d}x = 3$ $\left[ax + \frac{6k}{x}\right]_1^2 = 3$ $(2a + 3k) - (a + 6k) = 3$ $2a + 3k - a - 6k = 3$ $a - 3k = 3$ $a = 3 + 3k \quad (1)$ Also $\left[\frac{6x^2}{2}\right]_1^2 = a$ $[3x^2] = a$ $3k^2 - 3 = a$ Substituting eq (1) into this gives: $3k^2 - 3 = 3 + 3k$ $k^2 - k - 2 = 0$ $k = 2$ or $k = -1$	• Evidence of both correct integrations.	• Correct solution for both values of *k,* with evidence of correct integration.	
(d)	$\text{Area} = \int_0^{\frac{\pi}{6}} 4\sin 5x \cos 3x\,\mathrm{d}x$ $= \int_0^{\frac{\pi}{6}} (2\sin 8x + 2\sin 2x)\,\mathrm{d}x$ $= \left[\frac{-2\cos 8x}{8} - \frac{2\cos 2x}{2}\right]_0^{\frac{\pi}{6}}$ $= \left(\frac{-2\cos\frac{8\pi}{6}}{8} - \frac{2\cos\frac{2\pi}{6}}{2}\right) - \left(\frac{-2\cos 0}{8} - \frac{2\cos 0}{2}\right)$ $= -0.375 - (-1.25)$ $= \frac{7}{8} = 0.875 \text{ units}^2$	• Correct integral.	• Correct area, with evidence of correct integration.	

(e)	$y(1-x) = (1+x)\dfrac{\mathrm{d}y}{\mathrm{d}x}$ $\displaystyle\int \frac{1}{y}\,\mathrm{d}y = \int \frac{1-x}{1+x}\,\mathrm{d}x$ $\displaystyle\int \frac{1}{y}\,\mathrm{d}y = \int -1 + \frac{2}{1+x}\,\mathrm{d}x$ $\ln y = -x + 2\ln(1+x) + c$ $\ln 3 = 0 + 2\ln 1 + c$ $c = \ln 3$ $\ln y = -x + 2\ln(1+x) + \ln 3$ $x = 2 \Rightarrow \ln y = -2 + 2\ln 3 + \ln 3$ $\ln y = -2 + 3\ln 3$ $y = \mathrm{e}^{-2+3\ln 3}$ $y = \dfrac{27}{\mathrm{e}^2} = 3.654$	• Evidence of one of the correct integrations.	• Evidence of both correct integrations. AND Evaluation of the *c*-value.	**E7** Finding the value of ln *y*. **E8** Correct value for *y* found.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t	2t

	Expected coverage	**Achievement (u)**	**Achievement with Merit (r)**	**Achievement with Excellence (t)**
TWO (a)	$-(2x+1)^{-5}+c$ OR $\frac{-1}{(2x+1)^5}+c$	• Correct answer.		
(b)	$p=\frac{5}{4}\sin 4t+c$ $p=8, t=\frac{\pi}{24} \Rightarrow 8=\frac{5}{4}\times 0.5+c$ $c=\frac{59}{8}=7.375$ Then $p=\frac{5}{4}\sin 4t+\frac{59}{8}$ i.e. $p=1.25\sin 4t+7.375$	• Correct solution, with evidence of correct integration.		
(c)	$\frac{1}{2}\left[(4x+1)^{\frac{1}{2}}\right]_0^k=1$ $\frac{1}{2}\left[(4k+1)^{\frac{1}{2}}-1\right]=1$ $(4x+1)^{\frac{1}{2}}-1=2$ $(4x+1)^{\frac{1}{2}}=3$ $4k+1=9$ $4k=8$ $k=2$	• Correct integration.	• Correct integration. AND • Evaluation of the k-value.	
(d)	$v=\frac{2.4}{-0.3}e^{-0.3t}-t+c$ $6=-8e^0-0+c$ $c=14$ $v=-8e^{-0.3t}-t+14$ $s=\frac{-8}{-0.3}e^{-0.3t}-\frac{t^2}{2}+14t+d$ $t=0, s=0 \Rightarrow 0=\frac{80}{3}e^0-0+d$ $d=-\frac{80}{3}$ $s=\frac{80}{3}e^{-0.3t}-\frac{t^2}{2}+14t-\frac{80}{3}$ $s=\frac{80}{3}e^{-0.9}-\frac{9}{2}+14t-\frac{80}{3}$ $s=21.675$ metres	• Correct formula for velocity, including evaluating the c-value.	• Correct distance of 21.675 metres found, with evidence of integration methods.	

(e)	Area $= \int_0^{\frac{\pi}{2}} \sin^3 x \cos^3 x \, dx$ $= \int_0^{\frac{\pi}{2}} \sin^3 x (1 - \sin^2 x) \cos x \, dx$ $= \int_0^{\frac{\pi}{2}} \cos x \sin^3 x - \cos x \sin^5 x \, dx$ $= \left[\frac{\sin^4 x}{4} - \frac{\sin^6 x}{6} \right]_0^{\frac{\pi}{2}}$ $= \left[\left(\frac{1}{4} - \frac{1}{6} \right) - 0 \right] = \frac{1}{12} = 0.0833 \text{ units}^2$ Alternative method: Area $= \int_0^{\frac{\pi}{2}} \sin^3 x \cos^3 x \, dx$ $= \int_0^{\frac{\pi}{2}} \sin x (1 - \cos^2 x) \cos^3 x \, dx$ $= \int_0^{\frac{\pi}{2}} \sin x \cos^3 x - \sin x \cos^5 x \, dx$ $= \left[\frac{-\cos^4 x}{4} + \frac{\cos^6 x}{6} \right]$ $= \left[0 - \left(-\frac{1}{4} + \frac{1}{6} \right) \right] = \frac{1}{12} = 0.0833 \text{ units}^2$		• Finds the correct integration, with clear methods demonstrated.	**E8** Finds the required area of 0.0833 units², with full justification.

NØ	**N1**	**N2**	**A3**	**A4**	**M5**	**M6**	**E7**	**E8**
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t	2t

	Expected coverage	Achievement (u)	Achievement with Merit (r)	Achievement with Excellence (t)
THREE (a)	Area $\simeq \frac{1}{2} \times 5 \times [2.12 + 1.88 + 2(2.32 + 2.65 + 2.54)]$ $= \frac{5}{2} \times [4 + 2 \times 7.51]$ $= \frac{5}{2} \times 19.02 = 47.55 \text{ m}^2$	• Correct solution.		
(b)	$= 5\int_1^k \frac{2}{2x-1}\,dx$ $= 5[\ln(2x-1)]_1^k$ $= 5[\ln(2k-1) - \ln 1]$ $= 5\ln(2k-1)$	• Correct answer.		
(c)	$\int (4y+1)^{-\frac{1}{2}} = \int x^{-2}\,dx$ $\frac{1}{2}(4y+1)^{\frac{1}{2}} = -\frac{1}{x} + c$ $y = 2, x = \frac{2}{3} \Rightarrow \frac{1}{2} \times 9^{\frac{1}{2}} = -\frac{1}{2/3} + c$ $\frac{3}{2} = -\frac{3}{2} + c$ then $c = 3$ $\frac{1}{2}(4y+1)^{\frac{1}{2}} = -\frac{1}{x} + 3$ $x = \frac{4}{5} \Rightarrow \frac{1}{2}(4y+1)^{\frac{1}{2}} = -\frac{1}{4/5} + 3$ $\frac{1}{2}(4y+1)^{\frac{1}{2}} = \frac{7}{4}$ $(4y+1)^{\frac{1}{2}} = \frac{7}{2}$ $4y + 1 = \frac{49}{4}$ $4y = \frac{45}{4}$ $y = \frac{45}{16} = 2.8125$	• Evidence of both correct integrations.	• Correct evaluation of the y-value, with evidence of integration methods.	
(d)	$\int_p^{2p} \frac{x^2+6}{x^4}\,dx = \frac{9}{4}$ $\left[\frac{x^{-1}}{-1} + \frac{6x^{-3}}{-3}\right]_p^{2p} = \frac{9}{4}$ $\left[-\frac{1}{x} - \frac{2}{x^3}\right]_p^{2p} = \frac{9}{4}$ $\Rightarrow \left[-\frac{1}{2p} - \frac{2}{8p^3}\right] - \left[-\frac{1}{p} - \frac{2}{p^3}\right] = \frac{9}{4}$ $\Rightarrow -\frac{1}{2p} - \frac{1}{4p^3} + \frac{1}{p} + \frac{2}{p^3} = \frac{9}{4}$ $\Rightarrow -2p^2 - 1 + 4p^2 + 8 = 9p^3$ $\Rightarrow 2p^2 + 7 = 9p^3$ $\Rightarrow 0 = 9p^3 - 2p^2 - 7$	• Correct integration.	• Proof completed, with evidence of integration methods.	

(e)	$\int_0^3 x\frac{1}{3}(x^2+3)^2\,dx = \left[\frac{(x^2+3)^3}{18}\right]_0^3$ $= \frac{1}{18}[12^3 - 3^3]$ $= \frac{189}{2} = 94.5$ $\int_0^3 \frac{1}{3}(x^2+3)^2\,dx = \frac{1}{3}\int_0^3 x^4 + 6x^2 + 9\,dx$ $= \frac{1}{3}\left[\frac{x^5}{5} + 2x^3 + 9x\right]_0^3$ $= \frac{1}{3}\left[\frac{648}{5} - 0\right]$ $\frac{216}{5} = 43.2$ Then $\overline{x} = \frac{94.5}{43.2} = 2.1875 = \frac{35}{16}$	• One integration found.	• Both integrations found.	**E7** Correct solution but with one minor error. **E8** Correct value of $\overline{x}$ found, with full justification.

NØ	N1	N2	A3	A4	M5	M6	E7	E8
No response; no relevant evidence.	ONE partial solution.	1u	2u	3u	1r	2r	1t	2t

Cut Scores

Not Achieved	Achievement	Achievement with Merit	Achievement with Excellence
0 – 6	7 – 12	13– 18	19 – 24