FIELD SCIENCES

Combined report for the Review of *Mathematics* and *Statistics and Probability* and the Review of *Mathematics (Mathematics with calculus)* Level 3 achievement standards

This review report was first published in November 2005.

In January 2006, a correction was made to **AS90636, Calculus 3.2.**

A mathematics teacher drew the attention of the Ministry to a minor error in the Level 3 Calculus achievement standard.

The achievement Explanatory Note, first bullet, second sub-bullet, included the phrase 'polynomials in factorised form'. This has been changed to 'polynomials in expanded form'.

In January 2006, Version 2 of the achievement standard was re-published and this report was amended and re-published to match the change made to the achievement standard.

Review of *Mathematics* and *Statistics and Probability* Level 3 achievement standards

Subfield Mathematics

Domain	Id	Subject reference
Algebra	90644, 90647	Statistics and Modelling 3.4, 3.7

Subfield Statistics and Probability

Domain	Id	Subject reference
Probability	90643, 90646	Statistics and Modelling 3.3, 3.6
Statistics	90641, 90642, 90645	Statistics and Modelling 3.1, 3.2, 3.5

The Ministry of Education and the Secondary Education Group Assessment business unit of NZQA have completed a review of the achievement standards listed above that were registered in October 2003.

New Registration date **November 2005**

Date new versions published **November 2005**

Planned review date **March 2008**

Summary of review and consultation process

This review is part of the planned cycle of review that takes place after the first year of use of the achievement standards.

- Initial feedback and comments were collated from teachers/schools, moderators, examiners, markers, material developers, and other stakeholders.
- This information was reviewed by a subject expert panel and recommendations were made in March 2005.

- The recommendations were sent out for full consultation to secondary schools, tertiary institutions, and others in the education sector, in April/May 2005.
- Feedback from this full consultation has informed the changes described below.

Main changes resulting from the review

AS90641, Statistics and Modelling 3.1

Achievement Criteria

- Achievement with merit – the criterion has been amended to 'analyse time series data to make a forecast' to clarify that students will make a forecast based on their analysis, using the trend and seasonal effects they have calculated from the data.
- Achievement with excellence – the criterion has been amended to provide a clear progression through the grades.

Explanatory notes

- For achievement and achievement with merit, the explanatory notes have been amended to clarify the requirements and reflect the feedback from moderation.
- For achievement with excellence, the first explanatory note has been removed as students are not intended to have to look at 'data that cannot be modelled by a single straight linear trend over its entire range' as the main focus. This is more appropriately covered as a discussion of improvements to the model. The second and third explanatory notes have been combined into one to allow greater flexibility in what the report can contain.
- At achievement with excellence, the focus on justified comments has been included to clarify the required standard and emphasize that an interpretation in context is important. 'Development and interpretation of an index number series' has been included as this is one way of checking the validity of the time series.

AS90642, Statistics and Modelling 3.2

- The words 'from large samples' have been added to the introductory statement, as this better describes the outcome of the whole achievement standard.

Achievement Criteria

- The achievement with excellence criterion has been amended to 'demonstrate an understanding of the theory behind confidence intervals' to better reflect the requirements at this grade and to show a sensible progression through the grades.

Explanatory notes

- For achievement, the 'difference between two means' has been added as it was considered to be appropriate for this grade.
- For achievement with merit, the words 'justifying or refuting claims about a population parameter' and 'estimating population totals' have been added to clarify what demonstrating understanding of confidence intervals involves.
- For achievement with excellence, it has been clarified that applications of the Central Limit Theorem could involve the distribution of sample means, distribution of sample totals or related probability problems involving the use of the normal distribution.
- General explanatory note 3 has been removed, as formal proofs are not required in this achievement standard.

AS90643, Statistics and Modelling 3.3

Explanatory notes

- For achievement with merit, it has been clarified that problems may involve interpreting probabilities or expected values.
- For achievement with excellence, it has been clarified that applications of probability theory may include linear functions of independent random variables.
- General explanatory note 2 has been amended to include ‘tables of counts or relative frequencies’.

AS90644, Statistics and Modelling 3.4

- The words ‘and interpreting solutions’ have been added to the introductory statement to indicate that a straight solution from a graphics calculator may not be appropriate and that some interpretation of this solution may possibly be required.

Achievement Criteria

- The word ‘or’ has replaced ‘of’ in the achievement with excellence criterion to clarify that the outcome or the process may be analysed or interpreted, as was the intention at this grade.

Explanatory notes

- The wording for achievement has been amended to make solving a system of linear equations optional. This will make assessment more manageable. Further information has been provided on the type of problems that may be used in assessment. The impact of graphic calculators on teaching and assessment has been considered. While no changes to the achievement standard are recommended as a result of this, the matter needs to be discussed/debated by the wider mathematics community.
- There was also consideration on the possibility of removing the non-linear methods but at this stage, the decision was made to leave them in, as iterative methods are an important mathematical process underlying much of the current technology.
- Solving a system of simultaneous equations has been made optional at achievement with merit to make assessment more manageable. The requirements for linear programming have been eased to make the standard for solving these problems match the others.
- General explanatory note 2 about derivatives of functions other than polynomials has been moved into the explanatory notes for achievement with merit, as this is where it applies.
- The words ‘constraints or objective function’ have replaced ‘parameters’ in achievement with excellence. This will clarify requirements for this grade.
- Deriving the formulae for the Newton-Raphson method has been removed from achievement with excellence, as this does not match the requirements for this grade.

AS90645 Statistics and Modelling 3.5

Title

- The title has been amended to reflect changes in the criteria.

Achievement Criteria

- The achievement criterion has been amended to 'select and analyse continuous bi-variate variables' to align it more closely with the relevant part of the curriculum that specifies investigating the relationship between two continuous variables rather than planning and carrying out an investigation. This amendment also reflects current assessment practice.
- The panel discussed the possibility of including discrete data in the achievement standard but decided against this, as it is not possible to do regression on discrete data.
- Achievement with merit has been amended to reflect movement away from the investigation as outlined above.
- Achievement with excellence has been amended to provide a clear progression through the grades.

Explanatory notes

- Achievement has been amended to reflect changes made to the criterion. It has been clarified that data should be appropriate for a linear model. Additional clarification on the requirements of the analysis has been provided.
- For achievement with merit, 'establishing a model using regression' has been removed, as this is included in the requirements for achievement. In addition, reference to 'regression analysis' has been removed, as it is also included in the requirement for achievement. Reference to 'coefficients of determination' has been included to provide an opportunity for comment on all aspects of the analysis.
- Achievement with excellence has been amended so that the focus is on justified comments and interpretation in context, which is the required performance at this grade.
- General explanatory note 3 has been amended to allow flexibility in the source of the dataset to be used for assessment.
- General explanatory note 4 has been inserted to ensure students have sufficient background knowledge to meet the requirements of the achievement standard.

AS90646, Statistics and Modelling 3.6

Explanatory notes

- Achievement has been amended to remove calculations of sample statistics, as this is considered more appropriate for achievement with merit. Also 'calculate probabilities using formulae' has been removed. It is no longer appropriate because of the use of technology.
- In achievement with merit, the use of normally distributed independent variables has been restricted to 'sums' and does not now include 'other linear combinations'. This better indicates the required level of performance for this grade.
- For achievement with merit the types of problems have been extended to include 'continuity correction where appropriate', 'combined events for Poisson, normal or binomial distributions', and calculations of sample statistics to estimate corresponding population parameters.

- The use of approximations has been removed from achievement with excellence, as this is no longer appropriate with the increased use of technology. 'Inverse Poisson' has been substituted for 'finding a parameter given a probability' to better clarify what is required at this grade.
- 'Linear combinations of normally distributed independent variables' has been included as this is more appropriate to achievement with excellence.

AS90647, Statistics and Modelling 3.7

Achievement Criteria

- The word 'practical' has been included in the achievement with merit criterion to reflect the curriculum, which requires choosing an appropriate model for real data.

Explanatory notes

- The words 'data will be raw data generated from a practical situation' have been included for achievement with merit to be consistent with the change in the criterion. The intention is that the students are expected to have sufficient knowledge of the data collection in order to be able to achieve all grades of this achievement standard.
- There has been some rewording of achievement with excellence to clarify that the focus is on justification within the context. The reference to 'deriving the formula for a logarithmic transformation' has been removed, as this is not required. However, 'relating the theory to the model' if it is within the context, could provide evidence for justifying the choice of model, and has been included.

Impact on Accreditation and Moderation Action Plan (AMAP)

None.

Impact on existing qualifications

None.

Impact of changes on NCEA Exclusions List

2005 Exclusions

Level 3 Achievement Standard	**Achievement or Unit Standard**
90644	5264

2006 Exclusions

Level 3 Achievement Standard	**Achievement or Unit Standard**
90644	5262, 5264

Summary of main changes to achievement standards' Ids, classification, titles, levels, and credits

The following summary shows the changes made to the achievement standards as a result of the review. All changes are in **bold**.

Key to review category

A Dates changed, but no other changes are made - the new version of the standard carries the same Id and a new version number

B Changes made, but the overall outcome remains the same - the new version of the standard carries the same Id and a new version number

C Major changes that necessitate the registration of a replacement achievement standard with a new Id

D Achievement standard will expire and not be replaced

Subfield Mathematics
Domain Algebra

Id	Title	Level	Credit	Review Category
90644	Solve equations	3	4	B
90647	Use a mathematical model involving curve fitting to solve a problem	3	3	B

Subfield Statistics and Probability
Domain Probability

Id	Title	Level	Credit	Review Category
90643	Solve straightforward problems involving probability	3	4	B
90646	Use probability distribution models to solve straightforward problems	3	4	B

Domain Statistics

Id	Title	Level	Credit	Review Category
90641	Determine the trend for time series data	3	3	B
90642	Calculate confidence intervals for population parameters	3	3	B
90645	Complete a statistical investigation involving bi-variate data **Select and analyse continuous bi-variate data**	3	3	B

Review of *Mathematics (Mathematics with calculus)* Level 3 achievement standards

Subfield Mathematics

Domain	Id	Subject reference
Algebra	90638, 90639	Calculus 3.4, 3.5
Calculus	90635, 90636	Calculus 3.1, 3.2
Trigonometry	90637	Calculus 3.3

The Ministry of Education and the Secondary Education Group Assessment business unit of NZQA have completed a review of the achievement standards listed above that were registered in October 2003.

New Registration date	**November 2005**
Date new versions published	**November 2005**
Date AS90636, Calculus 3.2 re-published	**January 2006**
Planned review date	**March 2008**

Summary of review and consultation process

This review is part of the planned cycle of review that takes place after the first year of use of the achievement standards.

- Initial feedback and comments were collated from teachers/schools, moderators, examiners, markers, material developers, and other stakeholders.
- This information was reviewed by a subject expert panel and recommendations were made in March 2005.
- The recommendations were sent out for full consultation to secondary schools, tertiary institutions, and others in the education sector, in April/May 2005.
- Feedback from this full consultation has informed the changes described below.

Main changes resulting from the review

AS90635, Calculus 3.1

Title

- The title has been amended so that it is consistent with the achievement criteria.

Achievement Criteria

- The two achievement with merit criteria have been combined into a single criterion to allow for greater flexibility in the assessment.
- The achievement with excellence criterion has been amended by removing the wording requiring ‘a combination of differentiation techniques’ to allow more variety in the problems that may be included. It was found that this requirement precluded the setting of some otherwise very suitable achievement with excellence type questions.

Explanatory Notes

Achievement

- Clarification is given that polynomials will be in expanded form.
- Differentiation using product and quotient rules for simple examples has been included.
- Reference to related rates of change has been removed as this is beyond the level expected for achievement. However, rates of change problems that may involve kinematics have been included.
- Finding equations of normals has been included to allow for more flexibility in assessment.

Achievement with Merit

- Sketching of a derived function from a graph underpins the understanding of the principles of differentiation and has been included.
- Examples have been added to clarify the required level of performance.

Achievement with Excellence

- Additional examples of the type of problems required have been added.

General Explanatory Notes

- Note 2 has been expanded to ensure that the candidates are aware that they must demonstrate differentiation skills.

AS90636, Calculus 3.2

Title

- The title has been amended to match the form of Calculus 3.1.

Achievement Criteria

Achievement

- The criterion has been amended by removing reference to methods of problem solving. The types of functions required have been included in the explanatory notes.

Achievement with Merit

- This criterion has been amended to more clearly indicate the progression from achievement.

Achievement with Excellence

- The criterion has been amended by removing 'a variety of integration techniques' to allow more variety in the problems that may be included. It was found that the requirement for a variety of integration techniques precluded the setting of some otherwise very suitable achievement with excellence type questions.

Explanatory notes

Achievement

- Clarification has been given that polynomials will be in expanded form and that diagrams will not necessarily be provided for areas and volumes.

Achievement with Merit

- This has been amended to provide a clearer indication that rates of change problems may be included where advanced integration techniques are required.
- The form of differential equation $y' = ky$ has been added for clarification.

General Explanatory Notes

- Note 2 has been expanded to ensure that the candidates are aware that they must demonstrate integration skills.

AS90637, Calculus 3.3

Achievement Criteria

Achievement with Merit

- The first criterion has been amended to clarify that modelling is an important aspect and that problems are required to be solved.

Achievement with Excellence

- The word 'more' has been added before 'complex' to clarify that complex numbers are not involved here.

Explanatory notes

Achievement

- The note has been amended to clarify that all trigonometric ratios may be involved and that the model will be given.

Achievement with Merit

- This has been amended so that candidates are now required to form the equation for the model and to clarify the level of difficulty and type of model expected at achievement with merit.

Achievement with Excellence

- Solving complex equations and 3-D trigonometric problems have been added to further clarify the type of problems to be expected.
- Clarification has been provided that evaluation of the model is appropriate only if the data has been collected by the candidates.

Some feedback on this achievement standard questioned the reason for it being internally assessed. The expert panel discussed this. The achievement standard is designated as internally assessed so that the candidate can have an extended period of time in which to develop and present their best evidence for the outcome being assessed. There is no expectation or requirement for this achievement standard that the assessment will involve practical work. The modelling is expected to be of a situation that is in a meaningful context for candidates.

AS90638, Calculus 3.4

Explanatory notes

Achievement with Merit

- The solution of $z^n = a + b\text{i}$ has been added to extend the selection of equations that require the use of De Moivre's theorem.

Achievement with Excellence

- The word 'extended' has been removed from the chain of reasoning statement to clarify the progression from this grade to scholarship (which does require 'extended reasoning').
- Notes have been added to clarify the progression from achievement with merit.
- Binomial expansions and loci (geometric representations of complex numbers) have been added to increase flexibility in assessment.

General Explanatory Notes

- Note 3 has been added to make the assessment more equitable for candidates who do not have graphics calculators.

AS90639, Calculus 3.5

Title

- The title has been amended to match the reviewed achievement criterion.

Achievement Criteria

Achievement

- The two criteria have been combined into a single criterion to make the assessment more manageable.
- 'Finding equations of conic sections' has been amended to 'writing equations related to conic sections' to increase the range of equations that may provide opportunities for evidence of achievement.

Achievement with Excellence

- This criterion has been amended to include 'more complex' to align with the excellence criteria in AS90635, Calculus 3.1 and AS90636, Calculus 3.2.

Explanatory notes

Achievement

- Examples of features have been added to clarify what is required.
- An indication has been added that at this grade the graphs of conic sections will be given when equations are required to be written.

Achievement with Merit

- This note has been re-ordered to emphasise the need for modelling of real situations.

Achievement with Excellence

- To clarify that mathematical contexts can be expected the reference to 'real life context' has been removed as this is required for achievement with merit.
- The word 'extended' has been removed to more clearly describe the progression to scholarship.
- Loci, directrix, and eccentricity have been added to clarify the scope of achievement with excellence.

General Explanatory Notes

- Note 2 has been added to emphasise the requirements of sketching.
- Note 3 has been modified to clarify the requirements for the use of technology.

Impact on Accreditation and Moderation Action Plan (AMAP)

None.

Impact on existing qualifications

None.

Impact of changes on NCEA Exclusions List

The following exclusion has changed.

2005 Exclusions

Level 3 Achievement Standard	**Achievement or Unit Standard**
90636	5266

2006 Exclusions

Level 3 Achievement Standard	**Achievement or Unit Standard**
90636	20660, 20905

Summary of main changes to achievement standards' Ids, classification, titles, levels, and credits

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Subfield Mathematics
Domain Algebra

Id	Title	Level	Credit	Review Category
90638	Manipulate real and complex numbers, and solve equations	3	5	B
90639	Sketch graphs and find equations of conic sections **Sketch graphs of conic sections and write equations related to conic sections**	3	3	B

Domain Calculus

Id	Title	Level	Credit	Review Category
90635	Differentiate and use derivatives to solve problems **Differentiate functions and use derivatives to solve problems**	3	6	B
90636	Integrate functions and solve problems by integration, differential equations or numerical methods **Integrate functions and use integrals to solve problems**	3	6	B

Domain Trigonometry

Id	Title	Level	Credit	Review Category
90637	Solve problems and equations involving trigonometric functions	3	4	B