Field   Sciences

Review of *Mathematics* and *Statistics and Probability* Level 3 achievement and unit standards

Unit standards

Subfield	Domain	ID
Mathematics	Algebra	5256, 5262, 5264, 5267, 5272
	Calculus	5265, 20660, 20905
	Geometry	20661
	Mathematical Processes	5254
	Mathematical Studies	5263, 7567, 12324, 12325, 12326, 12330
	Trigonometry	5268
Statistics and Probability	Probability	5258, 5259
	Statistics	5257, 5270, 7563, 11119, 12334

Achievement standards

Domain	ID	Subject reference
Algebra	90638	Calculus (C) 3.4
	90639	Calculus 3.5
	90644	Statistics and Modelling (S & M) 3.4
	90647	Statistics and Modelling 3.7
	90834	CAS Calculus (CAS C) 3.2
	90835	CAS Calculus 3.3
Calculus	90635	Calculus 3.1
	90636	Calculus 3.2
	90833	CAS Calculus 3.1
	90836	CAS Calculus 3.4
Probability	90643	Statistics and Modelling 3.3
	90646	Statistics and Modelling 3.6
Trigonometry	90637	Calculus 3.3
Statistics	90641	Statistics and Modelling 3.1
	90642	Statistics and Modelling 3.2
	90645	Statistics and Modelling 3.5

The Ministry of Education and NZQA National Qualifications Services have completed a review of the achievement and unit standards listed above.

New Registration date   **December 2012**

Date new versions published   **December 2012**

Planned review date   **December 2016**

Summary of review and consultation process

In 2008 the Ministry of Education (MoE) and NZQA began to review achievement and unit standards in light of the revised New Zealand Curriculum (NZC). This Alignment of Standards (AoS) review also addressed duplication of outcomes, credit parity, fairness, consistency, and coherence. The AoS review was guided by the revised NZC itself and the Standards Review Guidelines. A copy of the NZC is available at: http://nzcurriculum.tki.org.nz/Curriculum-documents/The-New-Zealand-Curriculum.

Teacher subject associations were involved in the review, and draft achievement standards were the focus of wide consultation, especially with secondary schools and teachers. Extensive resources, including student exemplars, were also developed to support these standards, and are available on the MoE and/or the NZQA websites.

The review of unit standards included consultation with tertiary providers to assess continued relevance and likely future use of the standards. Unit standards that duplicate achievement standard outcomes and those without the likelihood of future tertiary use were recommended for expiry.

National consultation was undertaken in 2011, with the results analysed by Research New Zealand. The responses were generally positive.

The review of these Level 3 unit and achievement standards was completed in time for implementation in schools in 2013.

Main changes resulting from the review

- All NZC Level 8 (NZQF Level 3) outcomes derived from the NZC are now assessed using achievement standards, and there are no longer any unit standards linked to the NZC.
- Existing achievement standards were reviewed and new achievement standards were developed to align with the NZC. See table below.
- Grading criteria for achievement standards were reviewed in accordance with the Standards Review Guidelines.
- Unit standards that recognised similar outcomes as achievement standards or that no longer aligned with the NZC were designated expiring. Unit standard 5268 was assigned a later expiry date (December 2015) to allow for managed transition of InfraTrain qualifications. See table below.

For a detailed description of the review of, and the changes to, the Mathematics and Statistics and Probability standards see the appendix at the end of this report.

Impact on existing organisations with consent to assess

Current consent for			Consent extended to		
Nature of consent	ID	Level	Nature of consent	ID	Level
Standard	5258	3	Standard	91585, 91586	3
Standard	5259	3	Standard	91585	3
Standard	5262	3	Standard	91574	3
Standard	5264	3	Standard	91574	3
Standard	5265	3	Standard	91578	3
Standard	5268	3	Standard	91575	3

Impact on Consent and Moderation Requirements (CMR)

All new achievement standards have been registered on CMR 0233.

Impact on registered qualifications

Key to type of impact	
Affected	The qualification lists a reviewed classification (domain or subfield) in an elective set
	The qualification lists a standard that has changes to level or credits
	The qualification lists a C or D category standard
Not materially affected	The qualification lists a standard that has a new title
	The qualification lists a standard that has a new classification

The following table identifies qualifications developed by other SSBs that are impacted by the outcome of this review. The SSBs have been advised that the qualifications require revision. The classifications and/or standards that generated the status *Affected* are listed in **bold**.

Ref	Qualification Title	ID	SSB Name
0672	National Certificate in Engineering and Technology (for the Design and Construction Sector) (Level 4)	**5256, 5262, 5264**	InfraTrain New Zealand
0453	National Diploma in Surveying (Level 6) with an optional strand in Mine Surveying	**5268**	

Impact of changes on Exclusions List

For transition purposes, the following exclusions will apply for new achievement standards.

Achievement standard	Excluded against each of these standards
91573 (3.1)	20661, 90639
91574 (3.2)	5262*, 5264, 90644*
91575 (3.3)	5268, 90637
91577 (3.5)	5267, 90638
91578 (3.6)	5265, 90635
91579 (3.7)	20660, 20905, 90636
91580 (3.8)	5257, 90641
91581 (3.9)	11119, 90645
91582 (3.10)	5270, 90642

91585 (3.13)	5258*, 5259, 90643
91586 (3.14)	5258*, 90646
91587 (3.15)	5262*, 90644*

Standards with an asterisk (*) appear in the table more than once.

Review Categories and changes to classification, title, level, and credits

The following summary shows the changes made to the standards as a result of the review. All changes are in **bold**. Where a new or a new version of an externally assessed achievement standard is registered, the following designation appears after the title **[Externally Assessed]**. Standards with an asterisk (*) appear in the table more than once.

Key to review category	
A	Dates changed, but no other changes are made - the new version of the standard carries the same ID and a new version number
B	Changes made, but the overall outcome remains the same - the new version of the standard carries the same ID and a new version number
C	Major changes that necessitate the registration of a replacement achievement standard with a new ID
D	Achievement standard will expire and not be replaced

Externally assessed achievement standards categorised as category C or D expire at the end of	December 2012

Unit standard 5268 expires at the end of	December 2015

Remaining internally assessed achievement standards and unit standards categorised as category C or D expire at the end of	December 2013

Sciences > Mathematics > Algebra

ID	Ref	Title	Level	Credit	Review Category
5256		Use sequences and series to solve problems in real and simulated situations	3	4	D
5262*		Use linear systems to solve problems	3	2	C
5264		Use numerical methods to solve problems	3	3	C
90644*	S & M 3.4	Solve equations	3	4	C
91574	**3.2**	**Apply linear programming methods in solving problems**	**3**	**3**	
5262*		Use linear systems to solve problems	3	2	C
90644*	S & M 3.4	Solve equations	3	4	C
91587	**3.15**	**Apply systems of simultaneous equations in solving problems**	**3**	**3**	
5267		Use complex numbers to solve problems	3	3	C
90638	C 3.4	Manipulate real and complex numbers, and solve equations	3	5	C
91577	**3.5**	**Apply the algebra of complex numbers in solving problems [Externally Assessed]**	**3**	**5**	

ID	Ref	Title	Level	Credit	Review Category
5272		Use functions and their graphs to model situations and solve problems	3	4	D
90647	S & M 3.7	Use a mathematical model involving curve fitting to solve a problem	3	3	D
90834	CAS C 3.2	Demonstrate an understanding of equations and expressions when solving problems	3	7	D
90835	CAS C 3.3	Demonstrate an understanding of patterns and relationships when solving problems	3	5	D

Sciences > Mathematics > Calculus

ID	Ref	Title	Level	Credit	Review Category
5265		Differentiate functions and use differentiation to solve problems	3	6	C
90635	C 3.1	Differentiate functions and use derivatives to solve problems	3	6	C
91578	**3.6**	**Apply differentiation methods in solving problems [Externally Assessed]**	**3**	**6**	
20660		Use integral calculus to solve differential equation problems	3	2	C
20905		Integrate functions and use integration to solve problems	3	5	C
90636	C 3.2	Integrate functions and use integrals to solve problems	3	6	C
91579	**3.7**	**Apply integration methods in solving problems [Externally Assessed]**	**3**	**6**	
90833	CAS C 3.1	Demonstrate an understanding of calculus concepts when solving differentiation and integration problems	3	7	D
90836	CAS C 3.4	Demonstrate an understanding of mathematical concepts without the use of electronic technology	3	5	D

Sciences > Mathematics > Geometry

ID	Ref	Title	Level	Credit	Review Category
91576	**3.4**	**Use critical path analysis in solving problems**	**3**	**2**	**New**

Sciences > Mathematics > Mathematical Processes

ID	Title	Level	Credit	Review Category
5254	Link mathematical processes and skills in problems	3	7	D

Sciences > Mathematics > Mathematical Studies

ID	Title	Level	Credit	Review Category
5263	Devise, use and evaluate algorithms to solve problems	3	2	D
7567	Plan, carry out and report on a mathematical study	3	3	D
12324	Apply and report on mathematical techniques necessary to design a system to meet given constraints	3	3	D
12325	Demonstrate understanding of recurrence relations and apply it in a variety of situations	3	3	D
12326	Demonstrate understanding of mathematical structure and apply it in a variety of situations	3	3	D
12330	Plan and analyse research on an aspect of mathematics	3	3	D

Sciences > Mathematics > Trigonometry

ID	Ref	Title	Level	Credit	Review Category
5268		Manipulate trigonometric expressions, sketch graphs of trigonometric functions, and solve equations	3	4	C
90637	C 3.3	Solve problems and equations involving trigonometric functions	3	4	C
91575	**3.3**	**Apply trigonometric methods in solving problems**	**3**	**4**	

Sciences > Statistics and Probability > Probability

ID	Ref	Title	Level	Credit	Review Category
5258*		Use expected values to solve problems	3	2	C
5259		Use probability models to solve problems	3	3	C
90643	S & M 3.3	Solve straightforward problems involving probability	3	4	C
91585	**3.13**	**Apply probability concepts in solving problems [Externally Assessed]**	**3**	**4**	
5258*		Use expected values to solve problems	3	2	C
90646	S & M 3.6	Use probability distribution models to solve straightforward problems	3	4	C
91586	**3.14**	**Apply probability distributions in solving problems [Externally Assessed]**	**3**	**4**	

* please note this standard appears in the table twice

Sciences > Statistics and Probability > Statistics

ID	Ref	Title	Level	Credit	Review Category
5257		Investigate time series	3	3	C
90641	S & M 3.1	Determine the trend for time series data	3	3	C
91580	**3.8**	**Investigate time series data**	**3**	**4**	

ID	Ref	Title	Level	Credit	Review Category
5270		Plan, carry out and report on an investigation to make inferences about a population	3	5	C
90642	S & M 3.2	Calculate confidence intervals for population parameters	3	3	C
91582	**3.10**	**Use statistical methods to make a formal inference**	**3**	**4**	
7563		Plan, carry out and report on a statistical investigation	3	3	D
11119		Investigate relationships between pairs of quantitative variables	3	4	C
90645	S & M 3.5	Select and analyse continuous bi-variate data	3	3	C
91581	**3.9**	**Investigate bivariate measurement data**	**3**	**4**	
12334		Demonstrate understanding of the application of statistics in quality management	3	3	D
91583	**3.11**	**Conduct an experiment to investigate a situation using experimental design principles**	**3**	**4**	**New**
91584	**3.12**	**Evaluate statistically based reports [Externally Assessed]**	**3**	**4**	**New**

Sciences > Mathematics

ID	Ref	Domain	Title	Level	Credit	Review Category
20661		Geometry	Sketch, describe and find equations of graphs	3	2	C
90639	C 3.5	Algebra	Sketch graphs of conic sections and write equations related to conic sections	3	3	C
91573	**3.1**	**Algebra**	**Apply the geometry of conic sections in solving problems**	**3**	**3**	

Appendix

Development of Level 3 Mathematics and Statistics Standards

Process of Aligning Standards with the New Zealand Curriculum

Most of the Level 8 Achievement Objectives have been included for assessment in a Level 3 standard.

Mathematics

Objectives that are not explicitly covered in the standards:

M8.2 *Display and interpret the graphs of functions with the graphs of their inverse and reciprocal functions.* Some examples of inverse functions are covered in the algebra and calculus standards (eg log is the inverse of exponential). Reciprocal functions are covered in the trig standard.

M8.4 *Use curve-fitting, log modelling, and linear programming techniques.* Only the linear programming is covered by Standard 3.2.

Statistics

The revised Curriculum has had a major change of focus in the Statistics strand. The emphasis is now on developing students' conceptual understanding to enable them to use statistics in a more realistic manner, aligned more to the types of analysis used in the real world. The selection of standards has been designed to support that intent.

There are three standards that involve investigations using the statistical enquiry cycle. Each standard reflects the different ways that analysis can occur: formal inference in comparing populations (3.10); finding and using models in time series analyses (3.8); finding and using models to investigate the relationship between two variables (3.9). The title of each of these standards reflects the type of investigation required.

Conducting experiments using experimental design is emphasised at curriculum Level 8 and a standard focusing on experimental design principles (3.11) has therefore been developed.

Another aspect of the new emphasis in the revised curriculum is the development of statistical literacy. AS 3.12 focuses on the students ability to critically evaluate a wide range of statistical reports to address this new emphasis.

Probability and probability distributions are an essential part of formal inference so the final two standards focus on this aspect of the revised curriculum. AS 3.13 focuses on the application of probability concepts while AS 3.14 focuses on the application of probability distributions.

Addressing Duplication

There is no duplication of achievement objectives within the draft aligned standards. Any unit or achievement standards that duplicate the new aligned standards have been designated expiring. The four CAS achievement standards have been designated expiring and are not being replaced. Similar outcomes in the NZC are now covered by the new

draft standards. Unit standards that no longer align with the NZC have also been designated expiring.

While the Statistical Investigation thread of the NZC contains two broad achievement objectives, the sub-bullets clearly indicate the different aspects of data analysis. These sub-bullets have therefore been focused on in considering the different standards as opposed to the broad achievement objectives.

Addressing Credit Parity

Credits were allocated based on learning and assessment time, using the general rule of approximately 10 hours of notional learning per credit.

Credit values have remained similar to the previous standards. The total credits now available at Level 3 are more than could reasonably be covered in one course. As with the Level 1 and 2 standards, outcomes from courses will need to be assessed by making an appropriate choice of standards from the matrix.

There are some small 2-credit standards remaining on the matrix. Consideration was given to combining these into larger standards but the design problem of having to assess two different Achievement Objectives in one standard proved too difficult. The advantage of retaining these smaller standards is that it makes credits more accessible to students, and allows more flexibility in designing courses.

External and Internal Assessment

Only three standards are externally assessed in each of Mathematics and Statistics.

The conics standard (3.1) is no longer externally assessed, which may allow for more flexible assessment options. New small standards for systems of simultaneous equations, critical path analysis, and linear programming are better assessed internally to give flexibility and accessibility.

In the standards derived from the Statistics strand, four of the standards are extended investigations and are therefore suited to be assessed internally. The other three standards are to be assessed externally.

The Design of the Standards

In accordance with the principles for the alignment of standards with the NZC project, the step up from Achieved to Merit to Excellence is described in terms of the quality of thinking required, not in terms of specific content.

The title of the standard is a clear and recognisable guide to the expected outcome. Explanatory Note 1 always contains the reference to the NZC document. Where this requires amplification additional detail is supplied in further Explanatory Notes.

The level of thinking expected for Achieved, Merit, and Excellence is defined in Explanatory Note 2. The first part of this definition describes the evidence required to achieve the standard. This is tailored to each standard but generally involves demonstrating knowledge of the relevant concepts and procedures in response to a problem, as well as communicating that evidence appropriately.

The descriptions of the evidence expected for Merit and Excellence are generic to all of the Level 3 Mathematics and two of the Level 3 Statistics standards. They are about the level of thinking, and are independent of the context or content. Two terms from the Solo Taxonomy have been used as appropriate generic phrases in the Achievement Criteria. These are "relational thinking" for Merit and "extended abstract thinking" for Excellence. These terms have been clarified in Explanatory Note 2 in ways intended to be meaningful to teachers.

The definition of the word "problem" is intended to clarify that the term covers problems set in a purely mathematical or statistical context as well as problems which require applications in "real-life" contexts.

The wording of the title as "--- in solving problems" is intended to convey that acceptable evidence could come from a partially successful solution to a problem. Communication of the process of solving a problem may yield the required evidence of thinking, even though a correct final solution to the problem is not obtained.

What Has Changed?

- The standards in the matrix are numbered in a way that makes progressions across the levels easier to see.
- Only three standards are assessed externally in each "subject".
- There are 30 credits available at Level 3 from the Level 8 Mathematics strand of the NZC.
- There are 28 credits available at Level 3 from the Level 8 Statistics strand of the NZC.

General Comments

Teachers are strongly advised to use the Teaching and Learning Guides at http://seniorsecondary.tki.org.nz, in particular the indicators and context elaborations, in conjunction with the standards to gain a deeper understanding of the specific content required for the teaching and learning programmes.

For a complete outline of replacement relationships, see table above.

Comments on individual draft standards

AS 3.1 *Apply the geometry of conic sections in solving problems*   (Internal, 3 credits)

This is the replacement standard for US 20661 and AS 90639.

In the teaching and learning of tangents and normals, note should be taken that implicit differentiation is not assessed in AS 3.6. This does not preclude teaching this in a programme of teaching and learning.

AS 3.2 *Apply linear programming methods in solving problems*   (Internal, 3 credits)

This is the replacement standard for US 5262, 5264, and AS 90644.

AS 3.3 *Apply trigonometric methods in solving problems*   (Internal, 4 credits)

This is the replacement standard for US 5268 and AS90637.

AS 3.4 *Use critical path analysis in solving problems*   (Internal, 2 credits)

This is a new standard has been developed to provide a progression at Level 3 that develops from the Level 2 standard on the use of networks.

AS 3.5 *Apply the algebra of complex numbers in solving problems*   (External, 5 credits)

This is the replacement standard for US 5267 and AS90638.

The intent of the standard has changed to better reflect the step up from Level 7 to level 8 of the NZC and the shift from real numbers to complex numbers. The title was therefore changed to better align with the standard.

The application of complex numbers to loci has been reinstated after sector consultation.

AS 3.6 *Apply differentiation methods in solving problems*   (External, 6 credits)

This is the replacement standard for US 5265 and AS90635.

Consultation feedback suggested splitting into standards with smaller credits. It was decided not to do this as it proved to be too difficult to define smaller discrete outcomes.

Content of the standard has been amended by removing implicit differentiation and differentiation from first principles to align the standard to the teaching and learning time required.

Competency at Level 7 NZC Mathematics strand is assumed and therefore references to these methods are not in the standard.

AS 3.7 *Apply integration methods in solving problems*   (External, 6 credits)

This is the replacement standard for US 20660, US 20905, and AS90636.

Consultation feedback suggested splitting into standards with smaller credits. It was decided not to do this as it proved to be too difficult to define smaller discrete outcomes.

Volumes of revolution is not in the NZC so this content will not be assessed in this standard.

Content of the standard has been amended by removing simple algebraic substitutions to align the standard to the teaching and learning time required.

AS 3.8 *Investigate times series data*   (Internal, 4 credits)

This is the replacement standard for US 5257 and AS90641.

Students are to select the time series variable to investigate from a multivariate data set and this is a change from the AS90641.

The interpretation of the time series is the primary focus. Technology is used for the analysis.

AS 3.9 *Investigate bivariate measurement data* (Internal, 4 credits)

This is the replacement standard for US 11119 and AS90645.

The title has been altered to emphasise that measurement data is required.

The use and interpretation of R^2 is not required at this level.

AS 3.10: *Use statistical methods to make a formal inference* (Internal, 4 credits)

This is the replacement standard for US 5270 and AS90642.

The credit value is 4 with randomisation assessed in AS 3.11; this reflects the teaching and learning time.

The Central Limit Theorem is not part of what is being assessed here, however it is part of the learning progression needed to understand a tool for formal inference.

Resampling methods are required to be used for making formal inferences.

AS 3.11 *Conduct an experiment to investigate a situation using experimental design principles* (Internal, 4 credits)

The title of this standard reflects the NZC and the emphasis on experimental design principles and the statistical enquiry cycle.

Randomisation is included in this standard.

AS 3.12 *Evaluate statistically based reports* (External, 4 credits)

This is a newly developed standard and is a progression from AS91266.

The title of the standard was changed to reflect the NZC and consultation feedback.

AS 3.13 *Apply probability concepts in solving problems* (External, 4 credits)

This is the replacement standard for US 5258, US 5259, and AS90643.

The descriptors for achieved, merit and excellence are different from the other statistics standards and reflect the style of the Mathematics standards as this standard requires problems to be solved as opposed to a statistical investigation.

AS 3.14 *Apply probability distributions in solving problems* (External, 4 credits)

This is the replacement standard for US 5258 and AS90646.

The main focus of this standard is on probability distributions as opposed to merely calculating normal, binomial, and Poisson probabilities to reflect the intent of the NZC.

EN 4 has been reworded to make this streamlined.

The descriptors for achieved, merit, and excellence are different from the other statistics standards and reflect the style of the Mathematics standards as this standard requires problems to be solved as opposed to involving a statistical investigation.

AS 3.15 *Apply systems of simultaneous equations in solving problems*

(Internal, 3 credits)

This is the replacement standard for US 5262 and AS90644.

The title of this standard reflects the relevant AO and this was supported by consultation feedback.